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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

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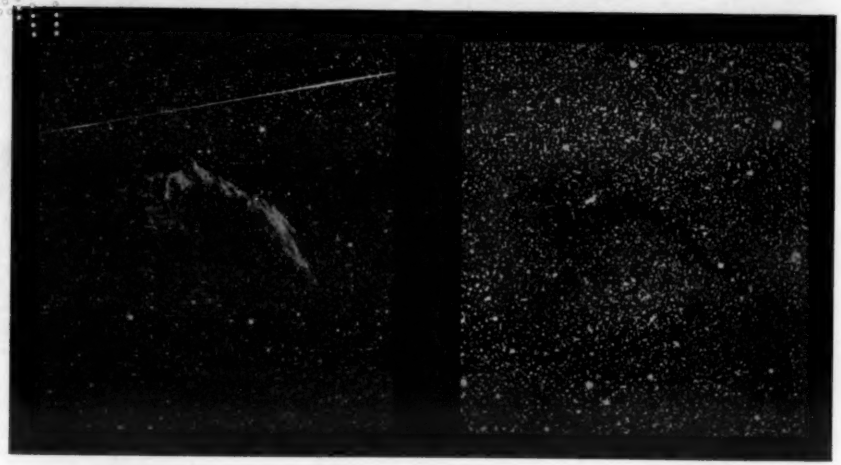
North



Gaseous Nebula N.G.C. 6995
1909 July 15. Exposure $5^h 43^m$

Dark Object in Cepheus
Shown on Plate II

Scale: 1 cm = 27′

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SOME OF THE DARK MARKINGS ON THE SKY AND WHAT THEY SUGGEST

By E. E. BARNARD

Our knowledge of the universe is confined to the stars and their attendant phenomena and to the nebulae. It is admitted that there are a great many dark stars, and there is even a claim that there are more dark than bright stars. This is entirely plausible, because we must assume that the stars ultimately die out or cease to give light. That they are everlasting, so far as their light is concerned, is more than improbable. If this is reasonable with respect to the stars, what can be said of the nebulae? If we can free ourselves from the belief that the function of a nebula is to become a star, or stars, we can imagine some similar condition to that of the stars to prevail among these great bodies, that is, that they may become dark in the course of time. Should this be true we shall have space peopled with bright and dark nebulae. Indeed, we shall have all the grades between a bright nebula and one entirely devoid of light.

Photographs of the sky often show us large dark markings comparable in size with the nebulae, scattered here and there over the heavens. They are found especially in the region of the Milky Way. The first impression these objects give is that they are

openings in the rich regions of stars through which we look out into the blackness of space beyond. The more familiar we become with them, however, the less this explanation appeals to us, and we soon begin to suspect that most of them are really dark or feebly luminous bodies shown in relief against a brighter background. Some of them are undoubtedly real vacancies, or places where the stars are actually thinned out, but these particular ones at once proclaim their nature and leave one in little doubt as to their explanation.

At present we are interested only in those dark markings that give the impression of being real objects and not simply vacancies. What their nature is we do not know, and the spectroscope cannot help us because the objects are devoid of light, or nearly so. But there is strong evidence that they are of the nature of the nebulae —that is, that they are dark nebulae.

Some of the dark stars make their presence known by their perturbing effect on the motions of bright stars, and by the eclipsing of their light. Their existence would otherwise be unknown, for their angular dimensions are too small for them to be seen in relief against any luminous background. Non-luminous nebulae, though their presence may never be known by any disturbance in neighboring bodies, would, if more or less opaque, be large enough to be seen if projected against a brighter background.

I think the general belief among astronomers is that a nebula remains luminous, and finally develops into a star or system of stars—that is, its ultimate destiny is a stellar condition. This opposes any supposition that a nebula may become dark by the loss of its light. For the presence of a dark nebula, however, it is not necessary that it should have lost its light. It may never have been luminous. It is possible that the original condition of a nebula is dark. It may be that some nebulae never become luminous, or it may still be possible that there are great non-luminous, opaque bodies in space that are not necessarily related to the nebulae, though this is perhaps far-fetched. From the fact that some of the nebulae appear to be partly dark and partly transparent, we might infer that they may, in the course of time, become either wholly bright or wholly dark.

In the case of Hind's variable nebula in Taurus, we have undeniable proof that a nebula may lose its light. Some fifty or sixty years ago this nebula lost all or most of its light, and from a conspicuous object in a small telescope, it ceased to be visible even in the most powerful instruments. Though it is again feebly visible in very powerful telescopes, it is mainly by photography that we can see it at all. What has occurred to this may happen to any other nebula, but the changes may take a much longer period of time.

I have shown that in certain cases a nebula may be opaque, or partly so. This is strikingly evident in the case of the great nebula about ν Scorpii and the one about ρ Ophiuchi.[1] In the case of the first of these nebulae it was shown that parts of it seemed to transmit the light of the stars behind it, and other parts—especially the darker or more obscure portions—seemed to cut out the light of the stars entirely. This possible opacity of a nebula leads us to a probable explanation of some of the smaller starless regions. That they are due to the interposition of masses of dark nebulosity seems more than probable, especially as long-exposure photographs frequently show feeble traces of nebulosity in these starless spots. A striking illustration of this is shown in the dark spot in Taurus (1855.0, $\alpha=4^h10^m$, $\delta=+27°55'$) where what appears to be a dark hole in the sky is exactly filled with excessively feeble nebulosity with dark details.[2] There are other equally striking examples of this connection of feeble nebulosity with vacancies. Perhaps a more remarkable example of this condition is that of the great nebula of ρ Ophiuchi and its immediate surroundings. All that is needed to make these dark bodies visible is a luminous region behind them. This is supplied in one way by the rich stellar regions of the Milky Way. An excellent example of how such a thing may be possible is shown by a phenomenon that presented itself to me one beautiful transparent moonless night in the summer of 1913, while I was photographing the southern Milky Way with the Bruce telescope. I was struck with the presence of a group of tiny cumulous clouds scattered over the rich star-clouds of Sagittarius. They were remarkable for their smallness and definite outlines—

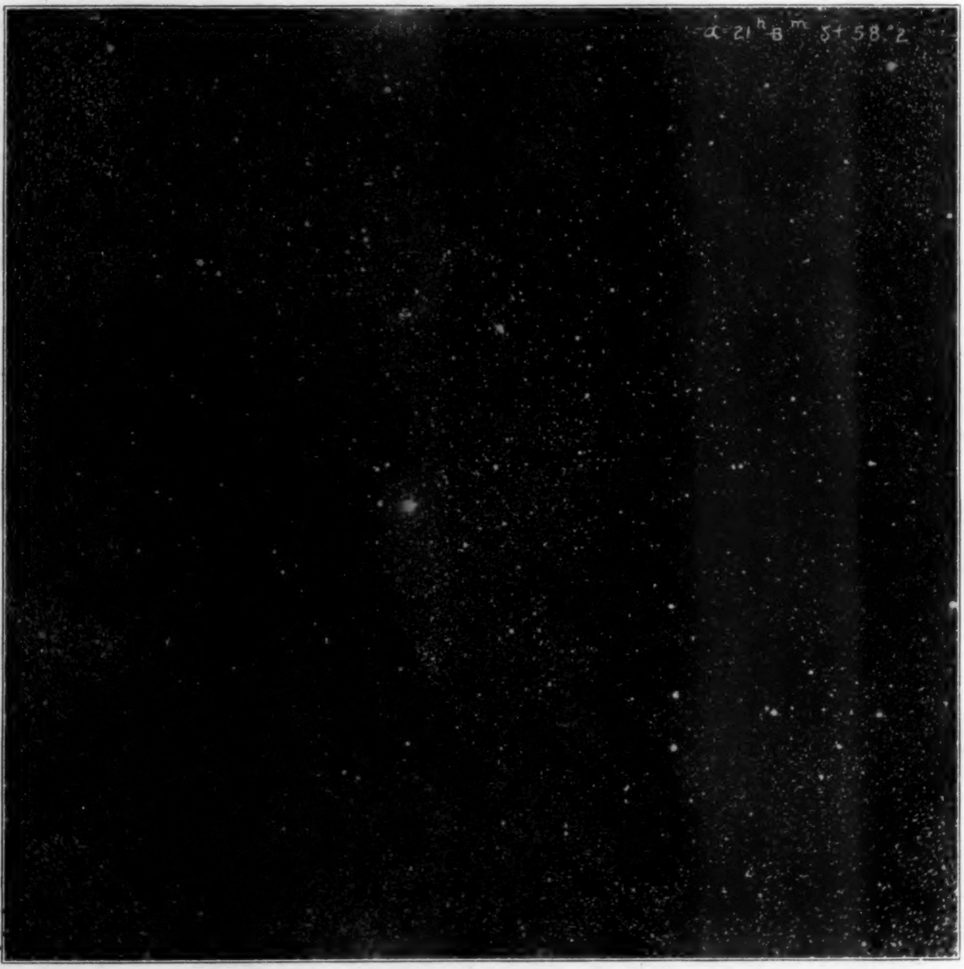
[1] *Astrophysical Journal*, **31**, 8, 1910. [2] *Ibid.*, **25**, 218, 1907.

some not being larger than the moon. Against the bright background they appeared as conspicuous and black as drops of ink. They were in every way like the black spots shown on photographs of the Milky Way, some of which I was at that moment photographing. The phenomenon was impressive and full of suggestion. One could not resist the impression that many of the black spots in the Milky Way are due to a cause similar to that of the small black clouds mentioned above—that is, to more or less opaque masses between us and the Milky Way. I have never before seen this peculiarity so strongly marked from clouds at night, because the clouds have always been too large to produce the effect.

Enough has been said, perhaps, to show that, whatever their condition may have been or may become, there are dark opaque objects scattered here and there over the heavens, which resemble, in all but light, the regular nebulae of the sky. Perhaps if we show a close resemblance in form and size of one of these to one of the well-known nebulae, it may aid us in connecting the two kinds of objects. For this purpose I have prepared Plate I from my photographs, which shows two different objects in different parts of the sky, one representing the bright nebulae and the other the "dark" nebulae. The photographs are on exactly the same scale and show that the two objects are of almost exactly the same angular size. The first is the nebula N.G.C. 6995 in Cygnus (1860.0, $\alpha = 20^h 51^m$, $\delta = +30^\circ 41'$), the other a dark marking recently found by me in Cepheus, in the position: 1860.0, $\alpha = 20^h 48^m$, $\delta = +59^\circ\!.6$. There is a striking resemblance in the forms of the two objects; but one is a luminous nebula and the other a dark—what? One can readily see that if the nebula were to lose its light, it would, if dense enough, still be shown against the sky and would strongly resemble the dark object. For this and many other reasons I am constrained to believe that the dark object is really a non-luminous nebula seen against a luminous background. There is no visible evidence of the ordinary nebulosity in the immediate region of this dark object; that is, the background is not the ordinary diffused nebulosity so frequently shown on photographs, nor is the stellar stratum dense enough to

PLATE II

North



REGION OF MILKY WAY SOUTH OF ALPHA CEPHEI, SHOWING TWO DARK OBJECTS AT POINTS INDICATED

10-inch Bruce Telescope. 1910 Oct. 1. Exposure $6^{h}2^{m}$

Center at $\alpha = 21^{h}8^{m}$, $\delta = +58\overset{\circ}{.}2$ (1855.0). Scale: 1 cm = $34\overset{\prime}{.}8$

serve as a luminous background. There is every evidence in the original photograph of a continuous, uniform luminosity over this entire region that is real and not due to the ordinary fogging of the plate by atmospheric luminosity. The idea has therefore suggested itself to me, and the suggestion has been strengthened by the appearance of other regions, that there is possibly a feeble luminosity in space, sufficiently strong to impress itself on the sensitive plate by prolonged exposures. I am, of course, familiar with the fact that our atmosphere is more or less luminous and that it fogs the plate with prolonged exposures, but palpably this could never serve as a background for the distant nebulae.

That the "dark" nebula of Plate I is between us and the stars in this region is clearly proved by the fact that it blots out the few stars that must be behind it. From an inspection of the original negatives it is clear that its visibility is not due to that fact, however, for the stars are too few at that point to serve as a luminous background.

That this luminous condition of the sky is widely extended in this region is shown by the presence of another similar, but much smaller, dark spot on the same plate, one degree south of α Cephei, which has a small star in it. This small object is close to and south of the 8.9 magnitude star B.D.$+61^{\circ}2903$. It is in the position 1855.0, $\alpha=21^{h}11^{m}\!.2$, $\delta=+61^{\circ}\!.8$, and is well shown on Plate II.

This last plate shows near its center a large fragmentary ring of nebulosity, part of which seems to be connected with a nebulous star, B.D.$+57^{\circ}2309(6^{M}\!.5)$. It lies in the position, roughly, $\alpha=21^{h}10^{m}$, $\delta=+58\frac{1}{2}^{\circ}$.

On the eastern edge of the plate is part of a remarkable nebula which is ramified with singular "dark lanes," which somewhat resemble the dark lanes in Taurus.[1] The nebulous star ($+57^{\circ}2309$) and nebulosities mentioned above were first shown on my plates with the Willard lens taken at the Lick Observatory.[2]

Two photographs were made with the 6-inch and 10-inch lenses of the Bruce telescope on September 30 and October 1, 1910, with

[1] *Astrophysical Journal*, **25**, 218, 1907.

[2] See *Publications of the Lick Observatory*, Vol. **11**, Plate 82.

exposures of 5^h5^m and 6^h2^m respectively. Plates I and II were made from the second of these negatives. I had hopes of making an entire night's exposure on this region, but the conditions have not been favorable. Such a long exposure was begun on October 9, 1915, with the dark object central on the plate, but after nearly six hours the sky suddenly clouded. The resulting negative verifies the other two, and shows that the marking is apparently on the sky itself, that is, it is not due to the presence of any ordinary nebulous background. Though this marking is purely a silhouette, and has no detail on its surface, there is much detail in the outline of its eastern end, which is more or less convoluted and has sharply defined projections from it like the horns of an insect. The body of it is much broken up.

If I have proved that there are dark objects in the heavens that are shown on photographs through being projected on a luminous ground I have opened the way to prove something else. This very fact of there being a luminous background may prove of the greatest value to us in our solution of the problems of space, because one form of this background suggests a feeble luminosity through interstellar regions and perhaps beyond.

If we assume that dark and opaque bodies exist in space, we could expect their presence to be shown by several means, all of which must be capable of producing a luminous background for the object. The kind of background will also give us some idea of the relative distances of these dark bodies. Such a body, if between us and the Milky Way, would appear black and conspicuous. We have many cases where such objects apparently exist in the Milky Way. One of the finest examples is that of the small black spot in a bright star cloud in Sagittarius (1855.0, $\alpha=18^h7^m$, $\delta=-18°17'$), which seems to be an opaque body in relief against the mass of bright stars. If one of these bodies were projected against a nebulous ground it would also be visible. We have examples of this, one being the dark spot in the nebulous strip running south from ζ Orionis,[1] which is probably a darker mass of the nebula itself, seen in relief against the more luminous portion of the nebula. Another beautiful example of this kind is shown in

[1] *Astrophysical Journal*, 38, 496, 1913.

photographs of the very elongated nebula 24 Comae[1] (N.G.C. 4565), which seems to be an object similar to the great nebula of Andromeda, with its edge toward us, where the darker outer periphery of the nebula is seen cutting across the brighter central region as a black irregular streak.

These examples are exceedingly interesting and important, because in the case of those objects seen against the Milky Way, it shows that these dark nebulae in many cases are nearer than the small stars that form the main body of the Milky Way. This in itself is not surprising to me, for we have many examples of bright nebulae that must be nearer than the more distant stars, such as the great nebula of ρ Ophiuchi, parts of which are also opaque nebulosity, and the nebula about σ Scorpii. Both of these are intimately connected with bright stars and must be relatively near to us. But there are others of these dark bodies that do not have a bright stellar background to show them, for they are seen in regions poor in stars, nor does their visibility depend upon the ordinary nebulous background, for there are no evidences of such nebulosity in the regions in which they appear. Without the stellar background or the presence of ordinary nebulosity these dark objects should not be visible. That they are distinctly visible against the sky under these circumstances would suggest, as their visibility would absolutely depend upon a more or less luminous background, that the sky beyond, for some reason, is itself luminous. If this uniform luminosity covered a very large region of the sky without any definite form or structure, there might arise a suspicion that distant space itself is more or less feebly luminous. There are a number of these dark spots whose presence is due to projection against a luminous background other than that produced by stars or nebulosity. They look like dark markings on the sky itself.

What is the cause of this luminous background where there is no ordinary nebulosity and where the stars are too few to produce it? There seems to be one possible explanation. Let us suppose space to be filled with the feeblest luminosity, so feeble, indeed, that at the distance of the fixed stars it is not sensible. Then, as space is supposed to be of very great extent, this luminosity would increase

[1] See W. S. Franks, *Monthly Notices*, **65**, 160, 1904.

in apparent density until it would finally become dense enough to affect the sensitive plate. This then would serve as a background to show in relief any large non-luminous object that might exist in space.

It is not necessary to attempt here any explanation as to the cause of this supposed luminosity of space. Indeed there is no direct claim that it actually exists. It is only suggested as a possible cause of certain phenomena that seem best interpreted by its presence—and from the further fact that the photographic plate seems to show its existence over large regions of the sky.

In respect to the illustrations: on the original negative the dark object is but feebly contrasted with the sky. To show it more clearly it was necessary, in order to gain contrast, to copy it repeatedly. This has produced the coarse grain on Plate I and gives the impression of a dense stellar background which only partly exists—the west portion of the object being projected on a much richer region of stars. The original plate is slightly fogged to the right of the dark nebula, which produces the luminous appearance close to the edge of the plate. The small plate is intended mainly to show the form of the object. The large one gives a truer representation of this region of the sky, with the exception of the slight fogging at the right side of the plate. The bright line on the photograph of N.G.C. 6995 is a large meteor trail.

An important fact that may come from our knowledge of the existence of dark nebulae is that their masses must be much greater than would be assumed for the ordinary nebulae, because they are perfectly opaque and must be relatively dense, and hence comparatively massive. If this is so, then, we must take into account these great masses in a study of the motions of the stars as a whole.

Yerkes Observatory
Williams Bay, Wis.
October 26, 1915

FACTORS AFFECTING THE RELATION BETWEEN PHOTO-ELECTRIC CURRENT AND ILLUMINATION

By HERBERT E. IVES, SAUL DUSHMAN, AND E. KARRER

I. INTRODUCTION

Experiments described in an earlier paper[1] showed that the relationship between illumination and current in potassium photo-electric cells as ordinarily constructed for photometric purposes is not rectilinear. No explanation was discovered for the extremely varied relationships found from cell to cell, but in general it was established that the relationship between illumination and current is a complicated function of applied voltage, electrode distance, and gas pressure. The investigations now reported have resulted in discovering the reason for the effects previously found, and in establishing the conditions necessary for a straight-line relation between illumination and photo-electric current. The various steps in the investigation will be described in the order in which they fall chronologically, for while some of them followed false scents, they furnished information which may answer questions arising in other connections.

II. EFFECT OF VARIATION OF GAS PRESSURE. CHARACTERISTICS OF HIGH-VACUA CELLS

The work previously reported included experiments on the effect of variation of the pressure of the gas within the cell. These showed gas pressure to be one of the significant variables. The new experiments fall under two heads: first, experiments on altering the pressure without opening or otherwise mechanically disturbing the tube; and, secondly, experiments with tubes with much better vacua than those previously used.

The experiments under the first head, in so far as they bear on effects of both illumination and voltage, were confined to a single

[1] Ives, "The Illumination-Current Relationship in Potassium Photo-electric Cells," *Astrophysical Journal*, 39, 428, 1914.

cell. This, lettered *d* in the previous paper, lent itself, because of its manner of construction, to a trial of the effect on the vacuum of a long-continued glow discharge. This discharge could be passed, as is evident from inspection of its construction (figured in the earlier paper), from the long platinum electrode to the guard-ring, without presumably affecting the potassium surface.

The cell was measured first before the passage of the discharge, both for the voltage-current and for the illumination-current relationships, the latter at two voltages, 261 and 174. It was then connected to a 5000-volt transformer, and a discharge passed until the vacuum reached the point where the current would no longer flow, at which point similar measurements were made. It was then connected to a small induction coil and run for a long period until no further discharge would pass. It was measured at this point, and then again after standing undisturbed overnight.

The four sets of measurements are shown in Fig. 1. Considering first the voltage-current curves, perhaps the most striking fact brought out is that the effect on sensibility of a gaseous atmosphere is altogether different at different voltages. Thus at voltages below 200 (with this particular cell) the effect of the gas is to inhibit the photo-electric current, notwithstanding the fact that the current is being assisted by ionization by collision, as is evident from the type of the voltage-current curves. It is evidently not permissible to describe a cell as being filled with gas "to the pressure which gives greatest sensibility," because this is one pressure for one voltage and another for a second. A further point of interest in connection with the voltage-current curves is that even at the best vacuum attained, saturation is not approached. This is presumably due, not alone to the imperfect vacuum, but to the inadequate anode area.

Before taking up the illumination-current relation in this cell, an additional and more complete set of data on the effect of gas pressure on sensibility may be reported.

The cell used for this study was similar in construction to those used by Elster and Geitel. It consisted of a 2-inch bulb with a platinum wire as an anode. The cathode was prepared by melting the ordinary potassium in a side tube which was connected to the

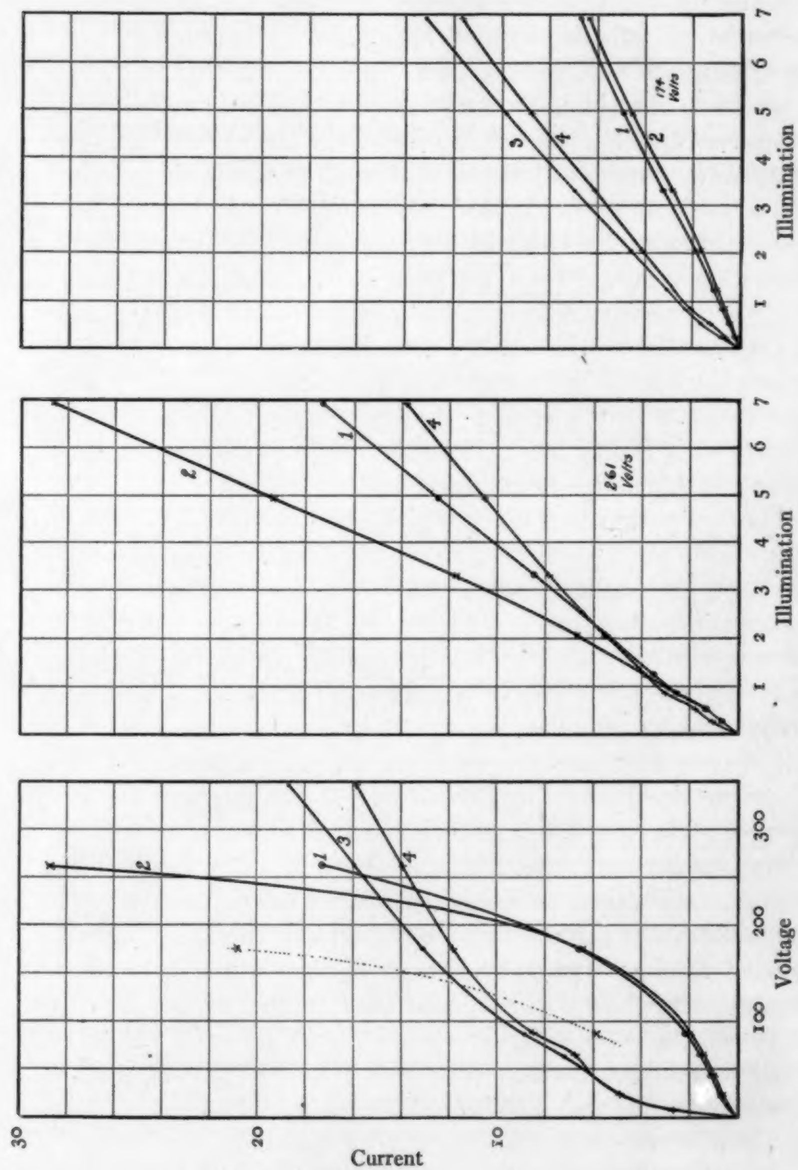


FIG. 1.—Characteristics of cell *d*, as pressure and other conditions were changed by the passage of a glow discharge between anode and guard-ring.

cell by means of a U-tube. The molten metal was distilled from adherent oxide into the cell under vacuum. The latter was connected by means of a mercury-seal stopcock to a McLeod gauge and reservoir containing purified hydrogen gas. A liquid-air trap was inserted between the cell and the rest of the system, thus preventing water vapor and carbon dioxide gas from entering the cell.

Observations were taken of the photo-electric current with constant illumination but different voltages. The source of the illumination was a 172 c.p. tungsten lamp placed about 4 inches above the potassium surface. The relative positions of the lamp and cell, as well as the voltage over the lamp, were maintained constant during the different sets of measurements. Fig. 2 illustrates graphically the results of these observations at different pressures of hydrogen gas. The numbers in parentheses indicate the order in which the observations were carried out.

The freshly prepared cell had a residual gas pressure of about 4 bar. This may have been either nitrogen or hydrogen gas—more likely the latter. The current-voltage curve obtained under these conditions is shown by (1), Fig. 2. It will be noted that the current varied only slightly with the voltage, but as the variation was linear, there was apparently no tendency toward a limiting current-value.

Hydrogen was allowed to enter the cell until a pressure of 1900 bar was attained, and the observations shown in curve (2) were obtained. The pressure was then reduced to 1380 bar and 600 bar in succession, as shown in curves (3) and (4). During the latter set of measurements an interesting effect was observed. The voltage applied to the cell was 160 at first and was then gradually increased to 220. The galvanometer deflection was observed to increase correspondingly from about 400 to 1450 divisions. On the changing of the voltage, however, from 220 to 240 volts, the photo-electric current increased at least tenfold and a glow appeared in the cell, indicating that a Geissler discharge had occurred. On the decreasing of the voltage now to 180, the galvanometer deflection was found to be 1700, that is, the sensitiveness of the cell had been increased considerably by the discharge. That this increase was not temporary is shown by the character of the subsequent curves.

Fig. 2, curve (5), gives the results of the observation obtained at 600 bar after the discharge. The general shape of the curves has remained unaltered, but the sensitiveness at the same voltage has increased in a ratio of approximately 2 to 1.

As the pressure of hydrogen is lowered from 260 bar to 1.9 bar, the current-voltage curves become flatter, so that (12) is quite similar to (1), but owing to the discharge previously mentioned, the photo-electric currents are over twice as great for the same voltage.

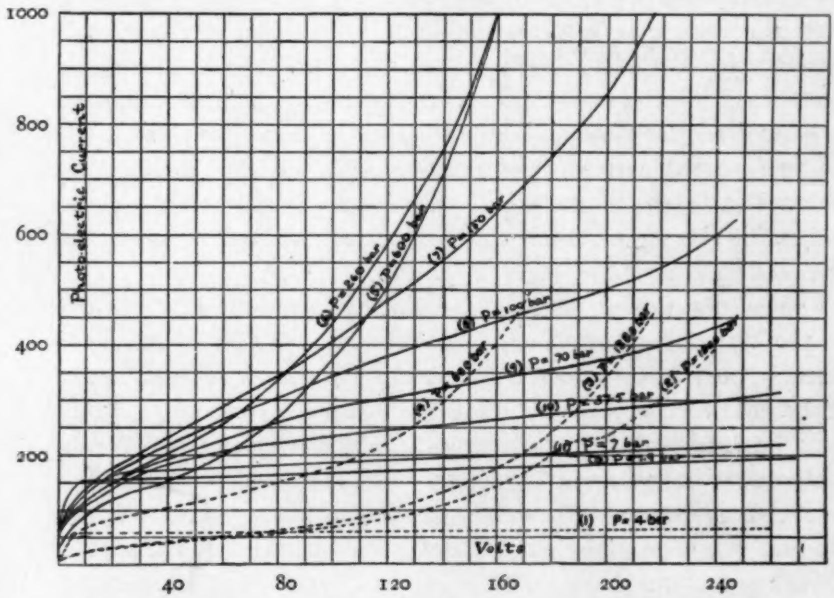


FIG. 2.—Voltage-current curves of cell, measured on pump, at various gas pressures. Dotted curves taken before passage of glow discharge, full curves after.

Any discharges, whether from a source of direct current at 240 volts (at higher pressures), or from an induction coil (at pressures of 1 to 2 bar), increased the sensitiveness of the cell, and usually there was an evolution of gas, as if the effect of the discharge consisted in decomposing some photo-electrically inactive compound on the surface. This hypothesis would also explain the subsequent "fatigue" which usually followed the increased sensibility. For, assuming a static equilibrium between the residual gas in the cell

and the gas present on the surface (either chemically combined or as an absorbed layer), the tendency would be for this equilibrium to re-establish itself after the discharge had ceased. That thin gas films inhibit the electronic emission from the surface of incandescent metals has already been shown pretty conclusively by the work of Langmuir,[1] and it appears quite probable that in the case of photo-electronic emission these effects would be even more pronounced.

Turning now to the illumination-current curves for cell *d*, a complicated state of affairs is found. At the lower voltage we have a complete transformation from curves that are convex to the illumination axis to curves that are concave. At the higher voltages similar changes occur, on which are superposed other smaller irregularities, and in addition, in agreement with the voltage-current curves, the relative sizes of the currents are different at the two voltages. Thus at 174 volts the order of increasing size of current is 2, 1, 4, 3; at 261 volts it is (neglecting the confusion near the origin) 4, 3, 1, 2.

These curves show in most convincing form how far from true is the unqualified statement that the photo-electric current is directly proportional to illumination. They show further, apparently, that the gas pressure, under the conditions present in certain cells, has an enormous influence on the actual relationship. We say *apparently* because, while the gas pressure was greatly changed, this change was also probably accompanied by changes in the amount of metal, grease, etc., in the cell walls, owing to the passage of the glow discharge. These latter changes, by altering the capacity of the cell walls to become charged, were, in view of our later work, probably the more important factors.

The experiments falling under the second head were undertaken in order to learn the character of the relationship of illumination to current when all gas is removed, as nearly as that condition can be approximated with available means of evacuation. Eight cells were constructed, differing from the previous ones in that the evacuation was performed with the Gaede molecular pump, in conjunction with a liquid-air trap to remove all grease and other vapors.

[1] Langmuir, "Thermionic Currents in High Vacua," *Physikalische Zeitschrift*, 15, 524, 1914.

The glass of the cells was baked at a temperature of 350° C. for some time previous to the introduction of the potassium, and the latter was distilled into place, usually in a number of stages, in one case (to be described later) as many as seven. In one cell, *p*, the vacuum was further improved by the process of burning out a tungsten filament after the cell was sealed off from the pump. The pressure of the residual gas in these cells is believed to be not more than 10^{-3} bar.

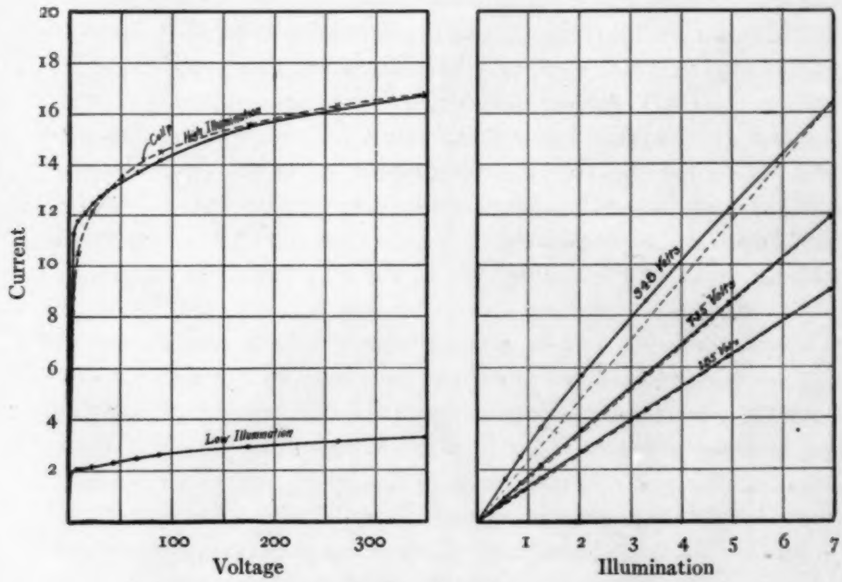


FIG. 3.—Characteristics of the typical high-vacuum cell (*m*)

These high-vacua cells differed from the earlier cells in several respects. They were all of considerably larger dimensions. While in the earlier ones the bulbs were only 3 or 4 cm in diameter, these were from 5 to 8 or 9, and in one case still larger. The anodes in these cells were either large loops of platinum wire, or, in the later ones, grids of tungsten wire. Guard-rings were usually dispensed with, as arrangements were made to work with comparatively large currents and low sensibility of the electrometer system.

Fig. 3 exhibits the characteristics of one of these cells, *m*. The voltage-current relation is characteristic of them all. The current

rises very rapidly up to about 10 volts, after which it mounts very slowly. Complete saturation is not reached at the highest voltages employed (350) in the electrometer measurements. Even at 5000 volts these cells have not shown complete saturation. The photo-electric current increases linearly with the voltage and apparently shows no tendency to approach a constant value. This failure to saturate may be due to several causes. The anode may not be sufficiently large. The grid electrodes used in several of the cells were intended to assist toward saturation, but, as is shown by the curve for cell *p* (Fig. 3), the improvement is little if, indeed, there is any. Again, there may be sufficient gas, or perhaps potassium or mercury vapor, present to prevent saturation. (The possibility of metallic vapor being present is, however, practically ruled out by the fact that surrounding the cell with carbon dioxide snow does not noticeably affect the voltage-current curve.) Probably, however, the most active cause is the reflection of electrons from the walls of the cell, which, as is well established, interferes with saturation.

Coming now to the illumination-current data, the curves of Fig. 3 are representative of all the cells only in certain features. It is characteristic of these cells that there are present none of the double curvatures and other local irregularities of the small cells. It is also characteristic of these cells that the greatest curvature is shown by the high-voltage curves.

But in the most significant of all characteristics, namely, the direction of curvature, the data on cell *m* are not representative for the reason that no two of these cells are alike. About half show illumination-current curves bending upward; the rest show curves bending downward. The certain elimination of nearly all the gaseous atmosphere and the reduction of all the cells to the same condition of vacuum, as shown by their characteristic curves, have not reduced them to uniformity in their behavior to light.

The extraordinary diversity of behavior of these cells is shown by the group of curves in Fig. 4. These, obtained (with the exception of cell *e*) from the new high-vacua cells, were measured upon a sensitive galvanometer, the currents being about one hundred times those worked with in the previous investigation. This shift

of instruments was made partly to make doubly sure that no instrumental peculiarities were figuring in the results, and partly because it was becoming evident that all the illumination-current peculiarities were accentuated with high voltages and currents.

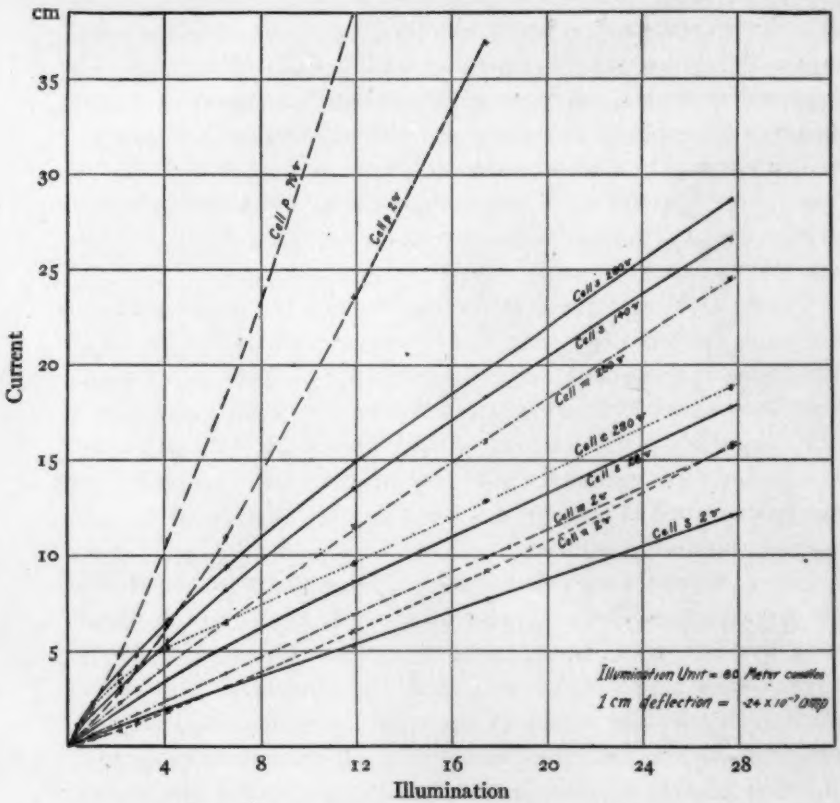


FIG. 4.—Illumination-current characteristics of a group of high-vacua cells

It is evident from these curves that the real cause of the non-rectilinear light-current relationship can hardly be the presence of gas. The striking effects of varying the gas pressure must be ascribed to the result of these changes on some other fundamental cause.

Further support for this belief was found in trying the effects of very intense illumination on cell *p*. If the gas present is a real

cause of the effects found, it is to be expected that if the applied voltage is held below the ionization voltage—that is, 6 or 7 volts—its influence should be minimized. By using a large nitrogen-filled tungsten lamp, illuminations up to 50,000 meter-candles were tried with only 2 volts applied, the currents being about fifty times those with the galvanometer previously employed, or up to 2×10^{-5} amperes (they were measured on a portable millivoltmeter). The illumination-current curve under these conditions was of exactly the same pronounced curvature as with the higher voltages and smaller currents. This experiment shows pretty clearly that no effects of gas ionization are active, as might have been thought from the fact that the illumination-current curves tended to flatten out with low voltages.

The greater smoothness of the curves from the high-vacua cells was suspected from the start to be in some way due to the larger dimensions of these cells, but the significance of this was not understood until several other clues were followed to their conclusion.

Before leaving the question of the behavior of high-vacua cells, an experiment, mentioned above, on multiply distilled potassium, may be described at length, because of its important bearing on the theory of photo-electricity.

Fig. 5 illustrates the manner in which this particular experiment was carried out. The apparatus used consisted of the tube *A* and bulbs *B*, *C*, *D*, *E*, *F*, *G*, *H*, and *K* in series. These were made of German glass and were about 2 inches in diameter. The bulbs *C* and *K*, while similar in size to the others, each had a small platinum wire sealed in at the bottom and a platinum loop from the side, thus making it possible to determine the photo-electric sensitiveness of the potassium, both after the first distillation as well as after the last. In addition to these electrodes, the cell *K* also had a platinum wire sealed in from the top, as indicated in the figure. The platinum wire used in making up these cells had been previously heated to near the melting-point in a good vacuum, in order to free it of dissolved gases. Between the last bulb and the molecular pump was inserted a liquid-air trap *T*. The dotted line indicates the oven in which the cells *C* to *K* were subsequently heated during exhaust. The end of the tube *A* was left open for

the introduction of the potassium (in the form of balls). A wad of glass wool was fitted in rather loosely at *g*, and after the potassium balls had been introduced into *A*, another plug of glass wool was inserted at *h* and the open end closed up with a torch. The potassium used was previously washed in ether and then dried between filter-papers.

After closing up tube *A*, the pump was started and the bulbs *C* to K heated for an hour at 350° C. Meanwhile the potassium in *A* was melted and allowed to run into *B*. During the bake-out of

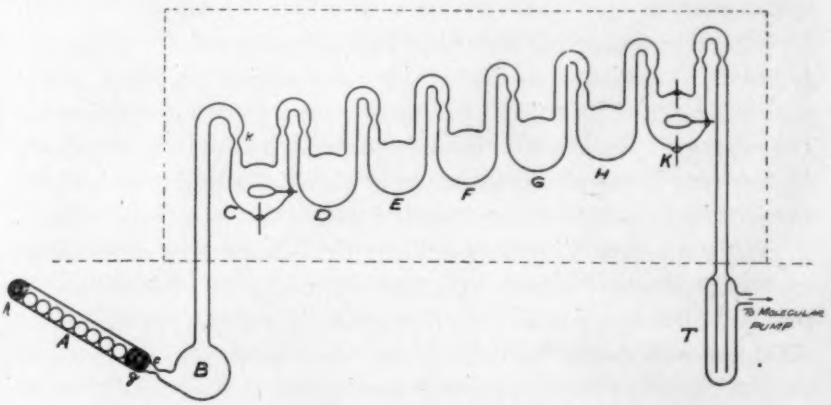


FIG. 5.—Arrangement of apparatus to investigate photo-electric effect from multiply distilled potassium in high vacuum.

the rest of the apparatus, the metal in *B* was heated gently, so that while it did not distil into *C*, it was yet kept hot enough for the gradual elimination of any gases that might be contained in it. At the end of the hour, the oven was raised and the metal distilled from *B* into *C*. Only about three-quarters of the metal in one bulb was distilled into the next. This permitted the gradual elimination of sodium which was undoubtedly present in the potassium.

After distillation into *C*, the bulb *B* was sealed off at *k*, and the photo-electric sensitiveness of the metal in *C* determined, while it was still in a molten condition. The potassium was then distilled in succession through the different bulbs, the latter being sealed off after most of their potassium content had been distilled

out. The whole operation, starting from the first distillation, took about three hours.

The results of this experiment indicated, first, that the photo-electric sensitiveness of the metal in C was not noticeably greater that that of the final distillation product; secondly, that the *photo-electric sensitiveness does not disappear when the metal is made as gas-free as possible, and the degree of vacuum is made as high as possible.* The illumination-current curve of this cell, made after its removal from the pump, is shown by n in Fig. 4. It differs neither in magnitude nor in character from that of the other high-vacua cells.

This experiment was described by one of the writers at a meeting of the American Physical Society in Washington, April, 1914,[1] and while one of its objects, as stated above, was to determine the character of the illumination-current relationship at extremely high vacua, it was also intended to settle another point which is of extreme importance for the theory of the photo-electric effect itself.

For some time there had been noticed a growing skepticism, especially among German investigators, as to the existence of a pure photo-electronic emission from metals in vacua. Fredenhagen,[2] Küstner,[3] and finally Wiedemann and Hallwachs[4] had come to the conclusion that "the presence of gas is an essential condition for the existence of the photo-electric effect."

Now, while the investigations of Langmuir and his associates had shown conclusively that there exists a pure electronic emission *per ipse* from heated metals in even the highest vacua, there was every reason for believing that the photo-electric effect also would continue to persist even in the highest vacua obtainable.

[1] See also Langmuir, "Thermionic Currents in High Vacua," *Physikalische Zeitschrift*, **15**, 524, 1914.

[2] "Das Ausbleiben des lichtelektrischen Effektes frisch geschabter Metalloberflächen bei völligem Ausschluss reaktionsfähiger Gase," *Physikalische Zeitschrift*, **15**, 65, 1914.

[3] "Das Ausbleiben des lichtelektrischen Effektes frisch geschabter Zincoberflächen bei völligem Ausschluss reaktionsfähiger Gase," *Physikalische Zeitschrift*, **15**, 68, 1914.

[4] "Dependence of the Photoelectricity of the Metal on the Gas, in Particular the Cause of the Strong Photoelectricity of Potassium," *Berichte der deutschen physikalischen Gesellschaft*, **16**, 107, 1914.

The experiment described above led us therefore to a conclusion which we had expected, but which is quite different from that arrived at by Wiedemann and Hallwachs.

Now what is the explanation of their results? The anode in their cell was a platinum wire sealed in from the top. As positive potential they applied about 8 volts; also before measuring the photo-electric current they passed a high-voltage discharge through the cell. Now we noticed in the case of cell K, when using the same positive voltages as Wiedemann and Hallwachs, *that a spark-coil brought near it caused the photo-electric current to decrease to zero.* Similar effects had been observed by Langmuir in measuring thermionic currents from heated filaments in highly evacuated bulbs, and the explanation suggested by him to the writers is as follows:

The rate at which electrons can reach the anode depends upon the voltage of the latter (space-charge effect) and upon the presence of neighboring charges. Any electrons striking the glass near the anode charge it up negatively, and in very high vacua there are not sufficient positive ions present to neutralize this charge; consequently it requires a very much higher positive potential on the anode to produce any noticeable photo-electric current. The discharge from an induction coil acts in a simlar way to charge up the glass around the anode, and with the low anode potential of 8 volts, the field near the anode is not sufficiently strong to neutralize the field, owing to the charges on the glass. However, on increasing the positive potential on the anode, or bringing a conductor near the glass so as to remove the charge, the photo-electric effect reappears and the cell acts normally.

With a given anode potential, the current to the upper electrode mentioned above in connection with the cell K was smaller than that to the loop, but the values tended to approach each other more and more as the voltage was made more and more positive.

Another point that ought to be mentioned in this connection is the observed effect of the presence of gases. It was noticed that traces of air, carbon dioxide, or water vapor sensibly decreased the photo-electronic emission from a potassium surface. On the other hand, the presence of hydrogen seemed to have no deleterious effect.

III. INVESTIGATION OF THE EFFECT OF CHANGED SURFACE CONDITIONS

Of the high-vacua cells, two were of particular interest, *p* and *s*. They both had grid anodes. Their voltage-current characteristics were the same. They were not greatly different in size, although one was approximately spherical, the other cylindrical. One of them, however, *p*, exhibited the most marked upward-bending light-current curve, the other, *s*, the most marked downward-bending curve of all the cells. They were therefore particularly suitable for crucial experiments.

They were carefully examined for differences, and the most striking one was found to be in their surface conditions. *p* had a very rough surface, evidently caused by quick cooling, while *s* had an almost mirror-like surface. The question arose whether the state of tension or annealing of the surface might not be of significance. The various shapes of the magnetization curves of iron and steel of different hardness offered counterparts for the different light-current curves, and while the phenomena are of course quite unrelated, it seemed worth while to investigate in the photo-electric cell the effect of a variable so important in the other case.

The test experiment was quite simple. Cell *p* was heated in an oil bath until its rough surface was molten, after which it was allowed to cool so slowly that it solidified as a smooth mirror. Cell *s* was similarly heated, and, when molten, was quickly taken to an open window and exposed to the outside winter temperature, being at the same time shaken so that its surface solidified into an irregular and diffusely reflecting form, closely resembling cell *p* before its transformation.

Both cells were then measured, with the very definite result that *their characteristics were absolutely unchanged*. Attention was then turned to another possible effect of surface conditions.

IV. INVESTIGATION OF THE NORMAL AND SELECTIVE EFFECTS

The known existence of two different photo-electric effects, the normal and the selective, differently localized in the spectrum, dependent on the character of the surface, the plane of polarization,

and the angle of incidence, offered another suggestive field for inquiry with respect to the light-current relationship.

In order to study the light-current curves of the normal and selective effects separately, the measuring apparatus was rearranged with the object of working with molten mirror-like surfaces, using light at oblique incidence. A separate sheet-iron box was provided in which the cells in a horizontal position could be heated in an electrically warmed oil bath. From this box well-protected wires ran to the electrometer and voltage supply. The light from a point-source tungsten lamp was arranged to fall at 45° through a nicol prism upon the potassium surface. The nicol prism was mounted to turn through 90° so that the light could be polarized either in, or at right angles to, the sensitive surface. The intensity of the illumination was altered by the use of sector disks, on the employment of which a word will be said presently. By means of this apparatus the obliquely incident polarized light can be received upon a molten mirror-like potassium surface. Under these conditions with the nicol turned one way the normal effect will be obtained; turned the other way, the selective.

With regard to the use of the sector disk, it is of course essential to establish by previous experiment that Talbot's law holds. Elster and Geitel have found this to be so, but it was thought desirable to perform their experiments again. Before dealing with this, however, it must be explained why recourse to the disks was found necessary. The standard method of working with polarized light where measurements of intensity are needed is by the use of two nicols, one of which is rotated. The present work was begun in this way, but it was soon found that with the pair of nicols used (which were old and not particularly well mounted) different quadrants gave different results. Thus with the analyzer, which was nearest the potassium, unchanged in position, the light-current curve obtained by rotating the polarizer from 0 to 90° was different from that obtained between 90° and 180°. The difference, though not large, was enough to invalidate the method for the present work. It is of course due to lack of parallelism of the incident light and slightly eccentric mounting of the nicol. These matters should be carefully looked into wherever nicol prisms are used for light-variation.

The test of the sector disk was made in a somewhat roundabout fashion, necessitated by the fact that the apparatus had been rearranged before the need for recourse to the disks was found. Fortunately one of the high-vacua cells was of such an intermediate character that for a certain voltage the light-current relationship was accurately a straight line. With this cell Talbot's law was found to be accurately followed. At the same time a measurement was made of the transmission of an opaque line grating on glass in a certain position. Another cell was then taken in which the illumination-current relation was not rectilinear and a set of points obtained using both the disks and the grating. The grating point fell accurately on the curve made by the disk points, proving that Talbot's law held for this cell as well, and hence presumably for any.

The experiments with the molten potassium surface were attended with considerable experimental difficulties, chief of which was the fact that the potassium begins to distil at its melting-point. This necessitated rapid working, and an alternation of direction of making observations, to eliminate the effect of the readings decreasing in magnitude by the clouding over of the cell walls. For the final conclusive measurements, when the general character was established, observations were confined to two points, for the low point either a disk or the grating being used.

As should be the case at this angle of light-incidence, the normal current was found to be much less than the selective. But, in the case both of cell *m* and of cell *p*, *the illumination-current relation is the same in the normal and in the selective effect.*

This experiment and that described in the previous section show that the surface differences between the two cells cannot be considered as the cause of their opposite behavior to varying illumination.

V. STUDY OF FOCUSING EFFECTS

The elimination of gas and surface effects as possible causes of the non-rectilinear light-current relationship made necessary some hypothesis along quite different lines. Effects due to the electron stream itself were next considered. First to be thought of were the effects of space-charge and of reflected electrons. The former

was not considered adequate, for one reason, because it would be excessively small with the currents obtained; for another reason, the most general effect of the space-charge would be to decrease the current below its normal values, as the number of electrons between the electrodes increased, while the experimental curves bend sometimes upward and sometimes downward. Reflected electrons were similarly thought to offer an inadequate explanation, for no reason was apparent why in some cases proportionately more and in other cases proportionately fewer electrons should be reflected as the number emitted was increased.

The clue which proved ultimately of most value was furnished by the idea of *focusing effects*. By this term is here meant a change of *direction* of the electron stream as the number emitted changes, whereby a different proportion of the whole number of electrons reaches the receiving electrode.

Examples of focusing effects are to be found in some of the early demonstration forms of cathode-ray tubes and in X-ray tubes. One of the former tubes, designed by Crookes to show the mutual deflection of two cathode streams, is an excellent example. Cathode rays from two adjacent, slightly inclined cathodes each normally proceed in straight paths through a common point of intersection. When, however, both streams are present together, they are mutually deflected, and their point of intersection is farther from the cathodes than the path of the individual undeflected streams would indicate. Were the current measured by a receiving anode, the current would increase more or less rapidly than the cause of the electronic emission, depending on whether the anode were inside or beyond the low-intensity focus of the two streams.

Another case of focusing is illustrated by the practice of placing the cathode of an X-ray tube back in a narrow glass stem. It has been found that this causes the cathode-ray stream to focus sharply and increase the hardness of the rays. This phenomenon is known to workers with discharge phenomena and has recently been again pointed out by T. Harris.[1] It has also been found that the position

[1] "On the Distribution of Electric Force in the Discharge at Low Pressures," *Philosophical Magazine* (6), 30, 182, 1915.

of the cathode in the neck of the discharge tubes has much to do with the intensity of the positive rays.[1]

In both cases the effect is ascribed to the presence of a strong electric field, in the first case in the cathode dark space, in the second on the glass walls of the cell, owing to the accumulation of a charge from stray electrons. These fields, varying with the strength of the current, and changing the direction or focus of the cathode-ray stream, offer satisfactory explanation of the peculiar effects found in the photo-electric cells. The ordinary photo-electric cell as constructed for photometric work, with its small dimensions, its large area of glass wall, and its straight wire or

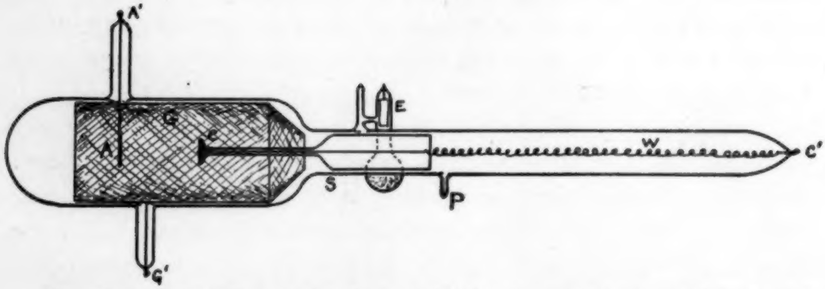


FIG. 6.—Special cell constructed to permit of various arrangements of anode and cathode. Total length, 45 cm; diameter of large part, 6 cm.

loop anode of small area, could hardly have been better designed for the purpose of producing an inextricable confusion of focused, deflected, and reflected electron streams. It is only necessary to put one of the small cells made in the earlier part of the investigation on an induction coil to get an idea, from the irregular streaks and splashes of fluorescence on the glass walls, of how complicated the conditions must be.

In order to test this hypothesis a special tube was constructed, shown in Fig. 6, designed to make possible a variety of different arrangements of anode, cathode, and surroundings. The cathode C is arranged to slide along the stem S; it is of glass, silvered at the end ordinarily lying in the body of the tube, the silver being connected to the sealed-in platinum wire C' by a short piece of platinum

[1] J. J. Thomson, *Rays of Positive Electricity*, p. 21.

wire through the glass, and a long coil of copper wire. In the process of making, *C* was pulled back to the side tube *P* from which the potassium was distilled on to the silver. *G* is a fine brass wire gauze, led to an outside connection *G′* and standing free from the glass walls. *A* is the anode, an iron plate of diameter 2 cm, with a separate outside connection *A′*. *E* is a charcoal evacuator, intended to assist the Gaede mercury pump which was used in this case. This cell was thoroughly baked during exhaustion, but, probably owing to the large amount of metal used in its construction, it did not have or maintain as good a vacuum as would have been desirable. This deficiency, in the light of the results obtained from the cell and later, was not serious.

Some nine different combinations of anode, cathode, and gauze connection, and cathode position, were tried with very interesting results.

Two of the voltage-current curves are shown in Fig. 7. Curve *a* is that obtained with gauze and anode connected to form a large inclosing anode, while the cathode is well up inside the gauze. Curve *d* is that obtained with the same connections by moving the cathode back into the stem. The latter curve was a somewhat difficult one to obtain for the reason that considerable time was required for conditions of equilibrium to be reached on deflection and return to zero. The behavior of the apparatus was entirely in accord with the idea that charging and discharging of the surrounding glass walls was taking place.

To obtain the illumination-current curves of this cell, the polarization apparatus was removed and a very short photometer track was improvised, on which was mounted a 100-watt point-source tungsten lamp. The movement of this lamp was between 12 and 35 cm from the diffusing glass, thus permitting the attainment of high illuminations and large currents, under which condition the most marked peculiarities are to be expected. At the same time the many contacts of the lamp filament and the unavoidable disturbances on moving it from point to point decreased the precision of the readings. The arrangement was therefore more suited to detecting changes of large order than for work of precision.

Fig. 8 shows four illumination-current curves. Curve *b* was obtained with A' and C' joined, and G as anode; current increases less rapidly than illumination. Curve *c* was obtained with G' and C' connected; current increases more rapidly than illumination. Curve *d* was obtained with cathode drawn back into stem; current increases less rapidly than illumination. Curve *e* is perhaps most interesting of all; it was obtained by letting A stand free, G

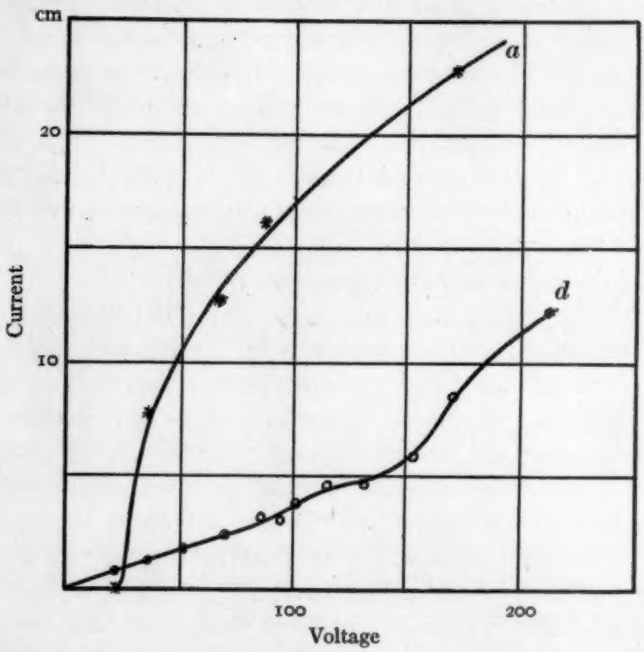


FIG. 7.—Voltage-current curves for two different positions of the cathode in special cell (Fig. 6).

serving as anode; here again the slow approach to final deflection and return to zero, and the tendency to stop and then start deflecting farther, clearly speak for the effect of charging and discharging of the isolated anode plate. The series of steps of Curve *e* are similar to those from cell *c* in previous work.

These results may be summarized in the statement that every type of illumination-current curve previously obtained has been duplicated in this one cell by varying the arrangement of its parts.

Curves *b* and *c*, representing the two types found in the high-vacua cells, find a ready explanation in the accumulation of charge on the side walls of the cell. The effect of this charge is to drive a continually increasing part of the current on to the plate *A*. This means an increasing current if *A* is the anode, a decreasing one if *A* is part of the cathode. In the case of curve *d* the charged walls

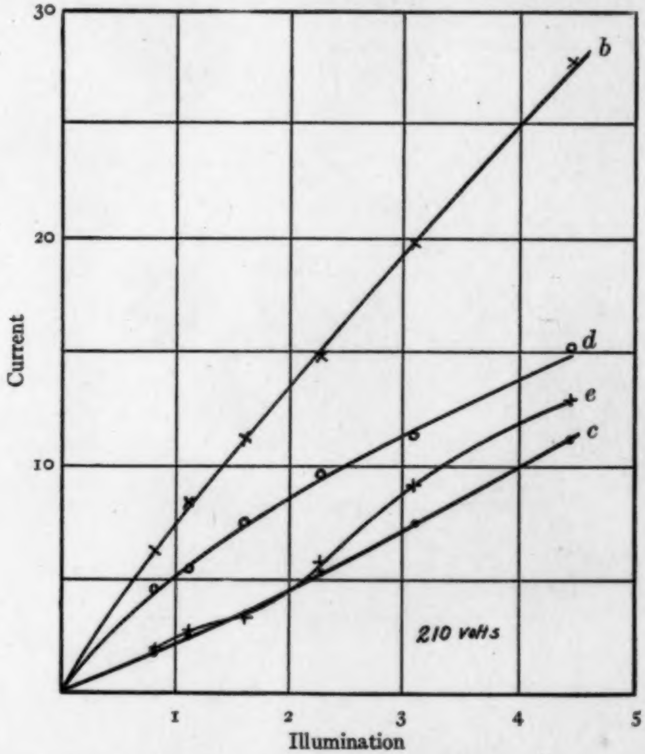


FIG. 8.—Illumination-current curves obtained with various electrode connections in special cell.

of the adjacent stem exert an inhibitory action on the emission of electrons. In the case of curve *e* the charged plate *A* is acting in opposition to the effect of the charged glass walls, with erratic results.

It appears, therefore, from our work that the accumulation of charges in the neighborhood of the photo-electric stream is a matter

of quite vital influence. Such charges if large may inhibit the photo-electric effect, as shown above, or they may seriously disturb the illumination-current relationship.

VI. THE TYPE OF CELL NECESSARY FOR A RECTILINEAR RELATIONSHIP

After finding by these experiments the cause of the erratic relationship between illumination and current which is likely to exist in cells as usually constructed, the next step was the design

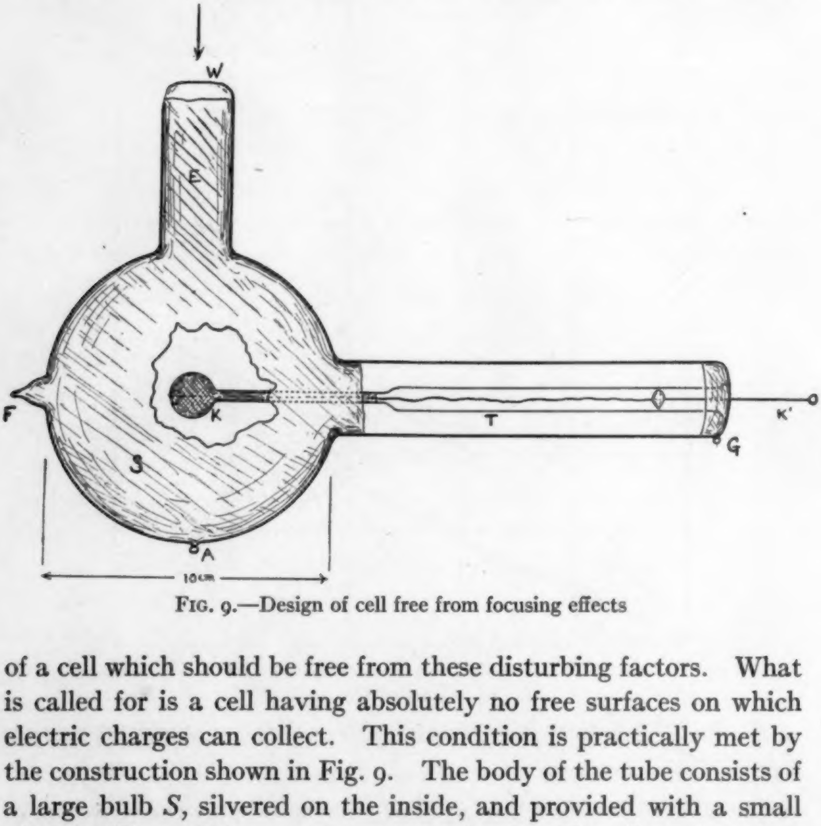


FIG. 9.—Design of cell free from focusing effects

of a cell which should be free from these disturbing factors. What is called for is a cell having absolutely no free surfaces on which electric charges can collect. This condition is practically met by the construction shown in Fig. 9. The body of the tube consists of a large bulb *S*, silvered on the inside, and provided with a small window *W*, carried on an extension tube *E*. The silvered surface serves as anode. The cathode is a small, centrally placed glass bulb *K*, which is silvered, and on which is distilled the alkali metal, which latter is first distilled on to the walls of the large surrounding bulb from a side tube. Wires at *A* and *K'* form the external points

of connection to anode and cathode. G is the guard-ring used to prevent conduction over the glass surface.

This cell will be recognized as identical in general form with that designed by Compton and Richardson,[1] whose object, however, was the elimination of electronic reflection. It is clear that with the exception of the necessary small illuminating window the cell is entirely symmetrical with respect to the cathode. Any tendency of the electron stream to focus on one part of the surrounding anode in preference to another is not to be expected, but should it occur the total current received by the anode is unaffected. Cells of this type should exhibit the true relationship between illumination and photo-electric current.

Tests made on three such cells have shown in each case, to within the limit of accuracy of the measurements, a truly rectilinear relation between illumination and current, under conditions of illumination and size of current which in the previous cells would have produced most marked deviations from such a relation.

An interesting point in connection with this design is that it is apparently a matter of indifference whether the cell is of the high-vacuum type or of the gas-filled type. One of the three so far made had a considerable atmosphere of gas, as is evident from the voltage-current curves exhibited in Fig. 10, but its behavior was like that of the others. There is indeed no reason why the presence of ionization by collision should introduce focusing effects in this type of cell. The evidence is that the effects on the illumination-current relation of changing the vacuum as was done in the experiment on cell *d*, described in the early part of this paper, are merely variations produced in the focusing effects which are inherent in the small cell with a single wire electrode, not effects produced directly by the gas. These variations in the focusing effects were due probably in part to the change in gas pressure, in part to the alteration in the insulating properties of the glass walls, owing to the "sputtering" of the platinum wire.

Strictly speaking, the three cells so far constructed are all to some extent gas cells, since their characteristic curves show that, in spite of the use of a charcoal evacuator in one case (X) and the

[1] "The Photo-electric Effect," *Philosophical Magazine* (6), **24**, 575–594, 1912.

molecular pump and liquid air in the other (*Y*), the best evacuated cells exhibit a second rise of current at about 20 volts which is undoubtedly due to ionization by collision. In these cells the effect of reflected electrons is presumably eliminated, so that the

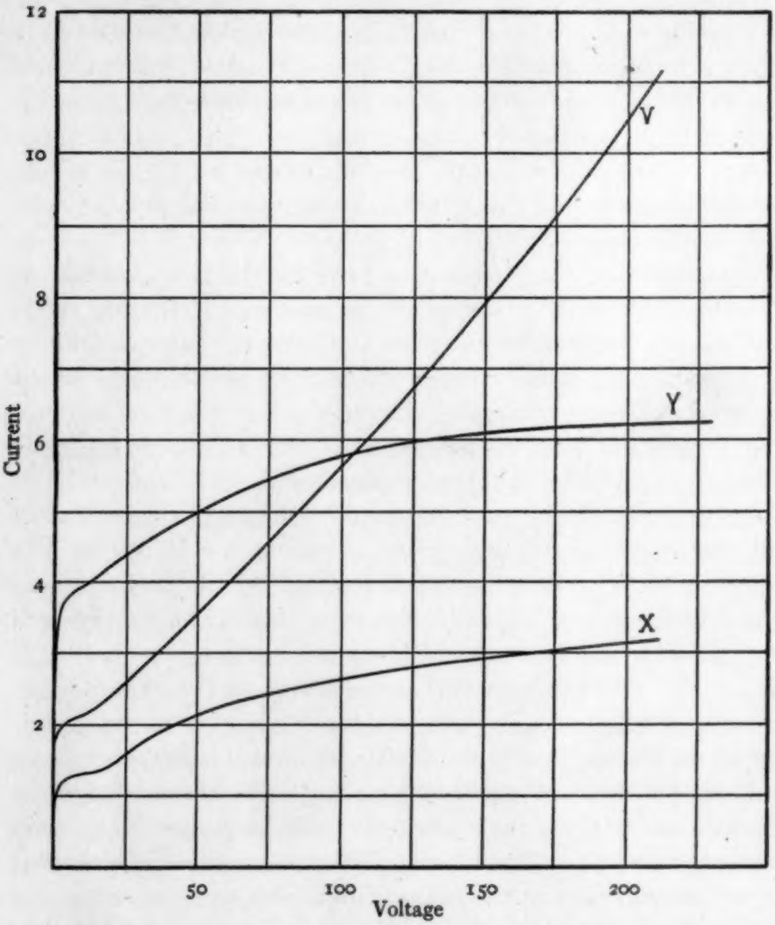


FIG. 10.—Voltage-current curves of three cells of the inclosing anode type

failure to attain a saturation current cannot be ascribed to them. Both *X* and *Y* show a faint glow on the induction coil (not shown before removal from the pump and therefore not affecting the conclusions with regard to the presence of the photo-electric effect

in a gas-free cell, which are supported by measurements on the pump). It is in fact our experience that the attainment of a saturation current in alkali metal cells working with high illuminations is a task of unusual difficulty. In spite, however, of the absence of a "perfect" vacuum cell from our tests, we feel warranted in the belief that cells of all degrees of evacuation, if constructed according to the design illustrated above, will give the much-desired straight-line relation between illumination and current.

VII. DISCUSSION

A question which demands answer is why these focusing effects have passed so generally unnoticed, in the widespread belief that *any* photo-electric cell will give a current proportional to illumination. It may be pointed out in this connection that here and there careful investigators have noted deviations from strict proportionality. Nevertheless the number of cases where the rectilinear relation has apparently been established is much larger than would be expected from the practically universal divergence from this condition in the cells of the usual form which were studied in this investigation.

The explanation probably lies in two facts: first, that most of the cells in use have gaseous atmospheres of several tenths of a millimeter, which may mask or actually reduce the focusing effects; secondly, that the common "dark current" due to conduction over the glass walls may be a blessing in disguise, in so far as it prevents the accumulation of charges on the walls. A larger portion of the time spent in the first portion of this research was devoted to the elimination of the spurious dark current. It now appears that the means which succeeded in preventing the dark current were peculiarly effective in making visible the focusing effects which had heretofore escaped notice. These effects now having been found and means discovered for combating them, we are in the condition of knowing definitely, not only how to produce a rectilinear illumination-current relation, but how to secure it free from undesirable companions. The advance is from a type of cell which would perform less satisfactorily the more carefully it was made to one which is based on definite knowledge of the conditions best calculated for all the qualities desired.

VIII. BEARING OF THE WORK ON STELLAR PHOTO-ELECTRIC PHOTOMETRY

In two previous papers[1] it was pointed out that the use of the photo-electric cell in photometry should be attended with great caution. Fro one thing, the relation between illumination and current was too uncertain to warrant the use of a cell without previous calibration. For another, no two of the curves of wave-length and sensibility appear to be alike. The first defect interferes with the cell's applicability to problems of variability of light, the second to its application to comparisons of one light-source with another of different color. In spite of these defects much work has recently been done with the photo-electric cell in stellar photometry, to which the extreme sensibility of the scheme commends itself.

The work here reported has an important bearing on stellar photo-electric photometry. It offers no hope that the varying color-sensibility from cell to cell may be avoided, but it does show how the first uncertainty above noted may be overcome. Moreover, the design shown in Fig. 7 is peculiarly suitable for stellar work, since in that work there is no advantage whatever in having a large area of alkali metal; the total amount of light receivable from the star is already concentrated into a small area by the telescope, and spreading it out does not increase the amount available.

A very important point in this connection is that this design offers possibilities for great sensibility, probably much greater than any yet used. By sensitizing the surface in the manner discovered by Elster and Geitel, and introducing an atmosphere of an inert gas, all the sensibility of their type of cell should be obtained. But, in addition to this a new aid to sensibility is offered, namely, an increase in the distance between the electrodes. From the theory of ionization by collision it follows that the *current in a gas increases rapidly with the distance between the electrodes.*[2] In the Elster and

[1] Ives, "The Illumination-Current Relationship in Potassium Photo-electric Cells, *Astrophysical Journal*, **39**, 428, 1914; "Wave-Length Sensibility Curves of Potassium Photo-electric cells," *ibid.*, **40**, 182, 1914.

[2] J. J. Thomson, *Conduction of Electricity through Gases*, p. 272.

Geitel form this fact cannot be utilized because of the small area of the anode. In the design here described it should be possible to make to advantage cells of very large diameter—how large would be a matter for experiment to determine. The glass stem of such a cell could easily be made of very great length, or it could be made perhaps of fused quartz, whereby, with use of a guard-ring, the dark current could be reduced by any desired amount. The complete design offers in fact possibilities in the way of sensibility and satisfactory performance which the writers cordially recommend as worthy of thorough study to those working in this field.

It is quite likely that the design of cell described by A. L. Hughes,[1] in which the walls are entirely covered with alkali metal, is equally satisfactory in its illumination-current relationship and will also increase in sensitiveness with size. Whether the increased sensitiveness of that type, due to the black-body conditions, is greater than that attainable in the inclosing anode type is a matter for experiment to decide.[2]

IX. SUMMARY

The occurrence of non-rectilinear relationships between illumination and photo-electric current is found to be due to focusing effects, caused by the accumulation of charges on the walls of the cells. A design of cell is given in which such effects are eliminated, whereby a truly rectilinear relation is obtained. It is suggested that cells of this type could be made which should exceed in sensibility and practicability any so far employed in stellar photo-electric photometry.

PHYSICAL LABORATORY OF THE
UNITED GAS IMPROVEMENT CO.
PHILADELPHIA
August 1915

[1] "A Sensitive Photo-electric Cell," *Philosophical Magazine* (6), **25**, 679, 1913.

[2] (Note added on correction of proof.) Experiments on one of our cells, on the walls of which considerable potassium remains, as a Hughes cell, indicate that this design gives the rectilinear illumination-current relationship and is several times more sensitive than the central cathode arrangement.

THE CHANGE OF COLOR WITH DISTANCE AND APPARENT MAGNITUDE TOGETHER WITH A NEW DETERMINATION OF THE MEAN PARALLAXES OF THE STARS OF GIVEN MAGNITUDE AND PROPER MOTION[1]

By P. J. VAN RHIJN

The present paper deals with the question of a possible relation between the color of the stars and their distances, apparent magnitude and spectral type being the same. It is found that the stars of the *Yerkes Actinometry*[2] show a relation between distance and color such that, *ceteris paribus*, the distant stars are redder than the nearer ones. Whether this phenomenon is to be ascribed to a loss of light in space or whether it is due to an influence of absolute magnitude cannot certainly be decided from the material available at present. Apart from its explanation, however, the question mentioned has an importance of its own, because its solution will afford a basis for further investigations; moreover, independently of the question as to how it is to be explained, a well-established relation between color and distance, apparent magnitudes and spectral types being the same, furnishes a valuable method for the determination of the distances of very remote objects.

For an investigation of the kind considered the *Yerkes Actinometry* contains very valuable material, viz., the visual magnitude, the color-index, and the spectral type of all the stars of the *Potsdam Photometric Durchmusterung* in the zone from 73° north declination to the pole. If the distances of these stars were also known, it would be a comparatively easy matter to derive the relation between color-index and distance. As, however, the parallaxes of only a few stars of the *Yerkes Actinometry* have been directly measured, we are obliged to use proper motion and apparent magnitude as a criterion of distance. I therefore derived the mean distance of stars of determined proper motion and magnitude, for which purpose I have made the following assumptions, in

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 110.

[2] *Astrophysical Journal*, 36, 169, 1912.

accordance with Professor Kapteyn[1] and Professor Schwarzschild:[2]

a) That the mean parallax of stars of determined proper motion and magnitude can be represented by the formula

$$\bar{\pi}=a\mu_1^b\epsilon^{m_1} \tag{1}$$

where $\mu_1=100\,\mu$ = total angular motion per century; $m_1=m-5.0$, m being the visual magnitude on the Harvard scale; a, b, ϵ are constants to be determined from the data of observation.

b) That the quantities

$$z=\log\frac{\pi}{\pi_0} \tag{2}$$

are distributed in accordance with the law of errors, π being the true parallax of a star and π_0 the most probable parallax of stars of the same magnitude and proper motion.

If the constants a, b, ϵ and the probable error ρ of the error-curve (2) have been determined from the data of observation, the mean distance of stars of given magnitude and proper motion can be derived by the formula

$$\bar{R}=\frac{1}{\bar{\pi}}e^{+\frac{\rho^2}{(0.6745)^2\ \text{mod.}^2}} \tag{3}$$

This is easily proved by means of supposition *b*).

The data from which the constants a, b, ϵ, and ρ must be determined are of two kinds: (1) the individual parallaxes; (2) the mean parallaxes of certain groups of stars, which are derived from the parallactic motions and radial velocities. As these constants may be different for stars of different spectral types, they have been derived separately for the B, A, and the second-type (F, G, K) stars.

Without entering into details as to the determination of the constants, the following may be stated:[3] The coefficients a and b were computed by means of the known individual parallaxes. For the helium stars I used those determined by Professor Kapteyn;[4]

[1] *Publications of the Astronomical Laboratory at Groningen*, No. 8, 1901.

[2] *Astronomische Nachrichten*, **190**, 361, 1912.

[3] The details of the whole investigation have been published in my thesis under the same title as the present note.

[4] *Mt. Wilson Contr.*, No. 82; *Astrophysical Journal*, **40**, 43, 1914.

for the F, G, and K stars, the parallaxes compiled by Professor Kapteyn and Dr. Weersma in 1910,[1] and those afterward published by the Yale Observatory,[2] by Flint,[3] Slocum and Mitchell,[4] and Abetti.[5] The parallaxes of stars of the same proper motion were combined into mean values which gave the data required for the determination of a and b. For the A stars this procedure was impossible, because the number of directly measured parallaxes of this type is too small to give trustworthy results.

The constant ϵ was determined from the mean parallaxes for stars of different magnitudes, derived from the parallactic motion and radial velocity. Only those stars were used whose radial velocities have been determined by Campbell.[6] For the second-type stars a second value of ϵ was derived from the directly measured parallaxes.

Finally the values of a and b derived from the individual parallaxes were corrected by means of the final mean parallax for stars of different magnitudes and proper motions, which had been computed from the radial velocities and parallactic motions. The final mean parallax of the A stars was used to determine the constant a, the coefficients b and ϵ being supposed equal to the average of the values found for the B and F, G, K stars. The constant ρ was determined from the differences between the logarithm of the observed individual parallaxes and the logarithm of the mean parallax computed by the formula (1). As the errors of observation in the parallaxes have a large influence on the value of ρ, I used only stars whose parallaxes are well determined.

Table I gives a summary of the results for the constants appearing in the formulae (1) and (3). The results are, in general, in agreement with those derived by Professor Kapteyn,[7] but the

[1] *Publications of the Astronomical Laboratory at Groningen*, No. 24, 1910.

[2] Chase, Smith, and Elkin, *Transactions of the Astronomical Observatory of Yale University*, **2**, 1912.

[3] *Astronomical Journal*, **27**, 49, 1912.

[4] *Astrophysical Journal*, **38**, 1, 1913.

[5] *Memorie del R. Osservatorio Astronomico al Collegio Romano*, Serie III, Vol. V Parte 2.

[6] *Lick Observatory Bulletins*, **6**, 108, 1911; **7**, 19, 1912; **7**, 113, 1913.

[7] *Publications of the Astronomical Laboratory at Groningen*, No. 8, 1901.

mean parallaxes of stars of different magnitudes found by equation (1), which are given in Tables II, III, and IV, are entirely at variance with those derived by Campbell.[1] The Lick Observatory

TABLE I

Type	a	b	c	ρ
B...........	0″0031	0.904	0.895	0.12
A...........	0.0028	0.80	0.895	0.15
F, G, K......	0.0038	0.695	0.895	0.17

results, however, cannot be considered to be representative of the whole stellar system, as the stars with large angular motion were omitted in deriving the mean parallaxes. It can be shown that the greater part of the difference between Dr. Campbell's parallaxes and those found here will disappear if the former are multiplied by certain factors which allow for the omission of the stars of large angular motion.

TABLE II

MEAN PARALLAX, TYPE B

μ	Visual Magnitude (Harvard Scale)										
	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
0″00....	0″0000	0″0000	0″0000	0″0000	0″0000	0″0000	0″0000	0″0000	0″0000	0″0000	0″0000
0.01....	0.0054	0.0048	0.0043	0.0039	0.0035	0.0031	0.0028	0.0025	0.0022	0.0020	0.0018
0.02....	0.0101	0.0090	0.0081	0.0072	0.0065	0.0058	0.0052	0.0047	0.0042	0.0037	0.0033
0.03....	0.0145	0.0131	0.0116	0.0105	0.0093	0.0084	0.0075	0.0067	0.0060	0.0054	0.0048
0.04....	0.0189	0.0170	0.0151	0.0136	0.0121	0.0109	0.0097	0.0087	0.0078	0.0070	0.0062
0.05....	0.0232	0.0207	0.0186	0.0166	0.0148	0.0133	0.0119	0.0107	0.0096	0.0085	0.0076
0.06....	0.0273	0.0244	0.0219	0.0195	0.0174	0.0157	0.0140	0.0126	0.0113	0.0100	0.0090
0.07....	0.0314	0.0281	0.0251	0.0225	0.0201	0.0180	0.0161	0.0144	0.0130	0.0115	0.0104
0.08....	0.0354	0.0317	0.0284	0.0254	0.0227	0.0203	0.0182	0.0163	0.0146	0.0130	0.0117
0.09....	0.0394	0.0353	0.0316	0.0282	0.0253	0.0226	0.0203	0.0181	0.0163	0.0145	0.0130
0.10....	0.0433	0.0388	0.0347	0.0310	0.0277	0.0248	0.0223	0.0199	0.0179	0.0159	0.0142
0.2.....	0.081	0.073	0.065	0.058	0.052	0.047	0.042	0.037	0.033	0.030	0.027
0.3.....	0.116	0.105	0.094	0.084	0.075	0.067	0.060	0.054	0.048	0.043	0.039

Having computed the mean distances of stars of given magnitude and proper motion, we pass on to the determination of the relation connecting color and distance. The color-index may also depend on the apparent magnitude, for we are not sure that

[1] *Lick Observatory Bulletin*, 7, 131, 1911.

the range of one magnitude is the same for both the visual and photographic scales. Therefore the equation for the color-index is

$$\sigma = a + bm + cR, \tag{4}$$

where

σ = color-index.
R = distance.
m = apparent visual magnitude.
b = change of color-index for a change of one unit in apparent magnitude.
c = increase of color-index per unit of distance (parsec).

TABLE III

MEAN PARALLAX, TYPE A

μ	Visual Magnitude (Harvard Scale)										
	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
0\".00....	0\".0000	0\".0000	0\".0000	0\".0000	0\".0000	0\".0000	0\".0000	0\".0000	0\".0000	0\".0000	0\".0000
0.01....	0.0049	0.0044	0.0039	0.0035	0.0031	0.0028	0.0025	0.0022	0.0020	0.0018	0.0016
0.02....	0.0085	0.0076	0.0068	0.0061	0.0054	0.0049	0.0044	0.0039	0.0035	0.0031	0.0028
0.03....	0.0118	0.0105	0.0094	0.0084	0.0075	0.0067	0.0060	0.0054	0.0048	0.0043	0.0039
0.04....	0.0148	0.0132	0.0119	0.0106	0.0095	0.0085	0.0076	0.0068	0.0061	0.0055	0.0049
0.05....	0.0177	0.0158	0.0142	0.0126	0.0113	0.0101	0.0091	0.0081	0.0073	0.0065	0.0058
0.06....	0.0204	0.0183	0.0164	0.0146	0.0131	0.0117	0.0105	0.0094	0.0084	0.0075	0.0067
0.07....	0.0231	0.0207	0.0185	0.0166	0.0148	0.0133	0.0119	0.0106	0.0095	0.0085	0.0076
0.08....	0.0257	0.0230	0.0206	0.0184	0.0165	0.0148	0.0132	0.0118	0.0106	0.0095	0.0085
0.09....	0.0283	0.0253	0.0226	0.0202	0.0181	0.0163	0.0145	0.0130	0.0117	0.0104	0.0093
0.10....	0.0308	0.0275	0.0247	0.0220	0.0197	0.0177	0.0158	0.0142	0.0127	0.0114	0.0101
0.2.....	0.054	0.048	0.043	0.038	0.034	0.031	0.028	0.025	0.022	0.020	0.018
0.3.....	0.074	0.066	0.059	0.053	0.047	0.043	0.038	0.034	0.031	0.027	0.024
0.4.....	0.093	0.083	0.075	0.067	0.060	0.054	0.048	0.043	0.039	0.034	0.031
0.5.....	0.111	0.100	0.089	0.080	0.071	0.064	0.057	0.051	0.046	0.041	0.037
0.6.....	0.129	0.115	0.103	0.092	0.083	0.074	0.066	0.059	0.053	0.047	0.042
0.7.....	0.146	0.130	0.117	0.104	0.094	0.084	0.075	0.067	0.060	0.054	0.048
0.8.....	0.162	0.145	0.130	0.116	0.104	0.093	0.083	0.075	0.067	0.060	0.053
0.9.....	0.178	0.160	0.143	0.128	0.114	0.102	0.092	0.082	0.074	0.066	0.059
1.0.....	0.194	0.174	0.155	0.139	0.124	0.111	0.100	0.089	0.080	0.072	0.064
2.0.....	0.338	0.303	0.271	0.242	0.217	0.194	0.174	0.156	0.140	0.125	0.111

The constants a, b, and c must be determined separately for each spectral type from a number of equations of the form (4). The color-index and visual magnitude were taken from the *Yerkes Actinometry;* the distance was computed for each star by means of formulae (1) and (3).

The solution involved two steps: First, the stars of each spectral class were divided into two or more groups according to their magnitudes. Forming the mean distance, color-index, and visual magnitude for all the stars of a group, a number of equations of the form

$$a + bm = \sigma + cR$$

were obtained, from which *b* was derived as a function of *c*. As the distances of the different groups were approximately equal, the constant *c* entered with a small coefficient. Secondly, the stars of each spectral class were arranged according to the magnitude of the proper motion. By means of these groups *c* was derived as a function of *b*, in which *b* entered with a small coefficient. From these two results, the values of *b* and *c* expressed in magnitudes were easily derived.

TABLE IV

MEAN PARALLAX, TYPES F, G, AND K

μ	Visual Magnitude (Harvard Scale)										
	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
0."00....	0."0000	0."0000	0."0000	0."0000	0."0000	0."0000	0."0000	0."0000	0."0000	0."0000	0."0000
0.01....	0.0066	0.0059	0.0053	0.0047	0.0042	0.0038	0.0034	0.0030	0.0027	0.0024	0.0022
0.02....	0.0107	0.0096	0.0086	0.0077	0.0069	0.0062	0.0055	0.0049	0.0044	0.0040	0.0035
0.03....	0.0142	0.0127	0.0114	0.0102	0.0091	0.0082	0.0073	0.0065	0.0059	0.0052	0.0047
0.04....	0.0174	0.0156	0.0139	0.0124	0.0111	0.0099	0.0089	0.0080	0.0072	0.0064	0.0057
0.05....	0.0203	0.0181	0.0163	0.0145	0.0130	0.0116	0.0104	0.0093	0.0084	0.0075	0.0067
0.06....	0.0230	0.0206	0.0184	0.0165	0.0147	0.0132	0.0118	0.0105	0.0095	0.0085	0.0076
0.07....	0.0256	0.0229	0.0206	0.0184	0.0165	0.0147	0.0132	0.0118	0.0106	0.0094	0.0084
0.08....	0.0281	0.0251	0.0225	0.0201	0.0180	0.0162	0.0144	0.0130	0.0116	0.0103	0.0093
0.09....	0.0305	0.0273	0.0244	0.0218	0.0196	0.0175	0.0157	0.0140	0.0126	0.0112	0.0101
0.10....	0.0328	0.0294	0.0263	0.0235	0.0210	0.0188	0.0169	0.0151	0.0135	0.0121	0.0108
0.2.....	0.053	0.048	0.043	0.038	0.034	0.030	0.027	0.024	0.022	0.020	0.018
0.3.....	0.070	0.063	0.056	0.050	0.045	0.040	0.036	0.032	0.029	0.026	0.023
0.4.....	0.086	0.077	0.069	0.061	0.055	0.049	0.044	0.040	0.035	0.032	0.028
0.5.....	0.100	0.090	0.081	0.072	0.064	0.058	0.052	0.046	0.041	0.037	0.033
0.6.....	0.114	0.102	0.091	0.082	0.073	0.065	0.059	0.052	0.047	0.042	0.038
0.7.....	0.127	0.114	0.102	0.091	0.081	0.073	0.065	0.058	0.052	0.047	0.042
0.8.....	0.139	0.125	0.112	0.100	0.089	0.080	0.072	0.064	0.057	0.051	0.046
0.9.....	0.151	0.135	0.121	0.108	0.097	0.087	0.078	0.069	0.062	0.055	0.050
1.0.....	0.163	0.146	0.130	0.116	0.104	0.093	0.083	0.075	0.067	0.060	0.054
2.0.....	0.263	0.236	0.211	0.188	0.169	0.151	0.135	0.121	0.109	0.097	0.087
3.0.....	0.348	0.312	0.279	0.249	0.223	0.200	0.179	0.161	0.144	0.129	0.115
4.0.....	0.426	0.381	0.341	0.305	0.273	0.244	0.219	0.196	0.176	0.157	0.140
5.0.....	0.498	0.446	0.399	0.356	0.319	0.285	0.256	0.229	0.206	0.184	0.164
6.0.....	0.565	0.506	0.453	0.404	0.362	0.324	0.290	0.260	0.233	0.208	0.186
7.0.....	0.628	0.562	0.503	0.450	0.403	0.360	0.322	0.289	0.259	0.232	0.207
8.0.....	0.690	0.618	0.553	0.494	0.442	0.396	0.355	0.318	0.285	0.255	0.227
9.0.....	0.748	0.670	0.600	0.536	0.479	0.430	0.384	0.344	0.309	0.276	0.246

Before proceeding to the actual solution of *c* as a function of *b*, it was necessary to correct the second members of the equations of condition for a systematic error, which tends to diminish the value of *c* for the early-type stars and to increase its value for the later types. The character of this error has been fully explained by Professor Kapteyn.[1]

[1] *Mt. Wilson Contr.*, No. 42; *Astrophysical Journal*, **30**, 293, 1909.

The value of b proved to be independent of the spectral type. We may therefore adopt for b the mean of the values found for the different spectral classes, viz.,

$$b = +0^m\!.025 \pm 0.004.$$

The values found for the constant c are given in Table V.

TABLE V

Spectrum	c	Probable Error
	Mag.	Mag.
B5 to A0	0.00000	±0.00006
A1 to A9	+0.00021	0.00006
F0 to G2	+0.00033	0.00007
G3 to K5	+0.00011	±0.00005

The weighted mean of these values is

$$c = +0^m\!.00015 \pm 0.00003.$$

The residuals with respect to the mean of the separate values in Table V show scarcely any systematic change with the spectral type. It may be that the B and A stars show the effect to a smaller degree than the later types, as is indicated by the value $c=0$ for the B5 to A0 stars. Mr. Adams' results,[1] derived by an entirely different method, give some indications in the same direction. Omitting the value for the B5 to A0 group, the mean for the other types is

$$c = +0.000195 \pm 0.00003,$$

a value 6.5 times its probable error.

CONCLUSION

The distant stars of the *Yerkes Actinometry* are, *ceteris paribus*, redder than the nearer ones. The increase of color-index per parsec is:

$$c = +0.00015 \pm 0.00003.$$

There are some indications that this effect does not exist for the B and early A stars. Supposing this to be the case, the amount of c for the other spectral types increases to

$$c = +0.000195 \pm 0.00003.$$

GRONINGEN
October 1915

[1] *Mt. Wilson Contrs.*, Nos. 78, 89; *Astrophysical Journal*, **39**, 89; **40**, 385, 1914.

ANOMALOUS DISPERSION AND FRAUNHOFER LINES

REPLY TO OBJECTIONS

BY W. H. JULIUS

If we realize the generally admitted gaseous nature of the sun, and the fact that nearly all of the solar radiation has passed through extensive, confusedly circulating masses of gas, we are inevitably led to consider the study of the propagation of light through such media as one of the necessary bases for the interpretation of the solar image and all its details. Some vigorous efforts recently made to prove that refraction has little influence on solar phenomena,[1] and that not much good should be expected from a broad application of the dispersion theory to astrophysical problems, can therefore only be regarded as a useful counterpoise against any excessive weight that might perhaps be given to the deductions from the theory of anomalous dispersion.

A thorough criticism of those deductions is certainly justified and welcome; for the remarkable versatility of the anomalous-dispersion theory renders the danger of exaggerating its application by no means imaginary, and the advocates of the theory should therefore always be kept on their guard against this danger.

The principal object of the present paper is to consider carefully the *arguments* of opponents to the idea that irregular refraction and anomalous dispersion probably are very important factors in the production of a great many solar (and stellar) phenomena usually ascribed to other causes.

[1] Anderson, *Astrophysical Journal*, **31**, 166, 1910; Gouy, *Compte Rendus*, **157**, 1111, 1913; Brunt, *Monthly Notices, R.A.S.*, **73**, 568, 1913; Evershed, *The Observatory*, **37**, 388, 1914; St. John, *Astrophysical Journal*, **41**, 28, 1915; *Mt. Wilson Contr.*, No. 93; St. John, *Proc. Nat. Acad. of Sciences*, **1**, 21, 1915; Adams and Burwell, *Astrophysical Journal*, **41**, 116, 1915; *Mt. Wilson Contr.*, No. 95; Adams and Burwell, *Proc. Nat. Acad. of Sciences*, **1**, 127, 1915.

In the present article consideration will chiefly be given to such arguments and subjects as can be discussed without entering into problems of theoretical optics; so we must defer the discussion of some of the objections raised by Gouy and Brunt to a later date.

As an introduction to this inquiry I wish to call attention to certain statements and assertions that have not the character of arguments, but are expressive of *opinions* regarding the astrophysical branch of the anomalous dispersion theory in general.

DISCUSSION OF OPINIONS

Some misapprehension seems to obtain with respect to the starting-point as well as the claims of that theory. This situation may be partly due to the use which is sometimes made of the term "optical illusion" when its fundamental notions are described. If, after having stated that the observed phenomena, such as prominences, flash spectra, flocculi, and displacements of Fraunhofer lines, are from this point of view mainly the effects of anomalous refraction in the solar atmosphere, an author continues, "so that in their study we are facing optical illusions,"[1] some readers will at once feel a little uneasy about the subject. No observer likes to be told that he is the victim of an optical illusion. The unfavorable mood is evoked, however, by an inadequate use of the term.[2]

In the case of a true "optical illusion" we are confronted with a difficulty in the appreciation of the relative dimensions, or positions, or perhaps colors of the parts of a visual image. The belief in the existence of a certain relation between the parts forces itself upon our minds, but, on closer examination, that relation proves not to exist. Our judgment thereby dealt with the *image* only; our decision greatly depended on our ability as observers; and we would rather not be led astray by the difficulties of estimation. The physical origin of the image is not involved in this question.[3]

[1] St. John, *Proc. Nat. Acad. of Sciences*, **1**, 21, 1915.

[2] I have once used the expression in the wrong sense myself (*Astron. Nachr.*, **160**, 141, 1902; *Physik. Zeitschr.*, **4**, 133, 1902), and regret the mistake.

[3] An example of a true optical illusion happens to be discernible on Plate I facing p. 19 of my paper in the *Astrophysical Journal*, **40**. The upper spectrum shows a few (five) sharp bright lines due to the carbon arc and alien to the interferential phenomenon with NO_2 under investigation. With respect to these bright lines the dark lines of the spectrum, which at first sight we believe also to be continuous straight lines, appear to be *inclined*.

The apparent dark lines, however, are built up of inclined sections, each shifted with respect to the next one (in consequence of anomalous dispersion), and we falsely impute the same inclination to the supposititious continuous straight lines.

If, on the other hand, various ***physical interpretations*** of a visible phenomenon are considered, the expression "optical illusion" for one of them is misplaced. Suppose two boys, both provided with some physical knowledge, had never learned anything about the sea, and are now for the first time looking from a distance at the breakers on the coast. One of them will say: "I see streaks and patches of a white liquid, like milk, appearing, moving, vanishing in blue-green water." "No," says the other, "I think it more probable that it is all water, and that the white patches are caused by reflection and refraction of the sky light in foam." As long as the boys cannot get on the beach, they will have to improve and discuss their observations and make theories; but neither of them can be rightly said to deal with an optical illusion.

Whatever may be the cause of the distribution of light in the visual images which we call sun-spots, prominences, etc., there must be a certain distribution of matter corresponding to those phenomena; so they are "real objects" in any case; and the displacement of a Fraunhofer line is a "real effect," whether it may be produced by motion in the line of sight, or by pressure, or by anomalous dispersion. The reliability of such phenomena as fundamentals of solar physics will not in the least be diminished by the necessity of considering them as dependent upon a physical process hitherto overlooked in their interpretation.

There may be some ground for presuming that the theory of anomalous dispersion "would revolutionize or render futile many of the present lines of solar and stellar observation"[1] because it might urge other questions to the front; but there need not be any fear that it "would make practically impossible the solution of many problems which confront the investigator";[1] the solutions may only turn out somewhat different from what they were expected to be from the points of view of other theories.

According to St. John there is "a degree of vagueness in the deductions from the theory, due to its extreme flexibility, that makes a quantitative examination of its claims difficult."[1] In my opinion, this theory has not "vagueness" for its own characteristic, but it has suggested that the bases of other theories may be more or

[1] St. John, *Proc. Nat. Acad. of Sciences*, **1**, 21, 1915.

less vague, for it has raised doubts as to the rectilinear propagation of light (especially of the so-called R-light and V-light) through the solar gases. The anomalous dispersion theory has then led us to investigate general and *regular* consequences of *irregular* refraction, and thus to endeavor to bring some order and clearness in what, at first sight, seemed to be a hopelessly entangled condition. One of its chief claims, therefore, is that it has combated vagueness.

Are there sufficient grounds for assuming the existence of such irregular gradients of optical density as the theory postulates? Adams and Burwell say: "Most solar observers would desire some evidence tending to indicate the existence of such gradients apart from the necessity of postulating them in order to support the anomalous refraction hypothesis, more especially as they must be essentially permanent in character."[1]

I was a little surprised to read this objection, because the desired evidence has been furnished in my paper "On the Interpretation of Photospheric Phenomena."[2] The main facts, arguments, and calculations there supplied are independent of the anomalous dispersion theory; and our conclusion that, in spite of the apparent sharp boundary, the sun may be gaseous until far below the photospheric level, and that scattering is the principal cause of the gradual darkening toward the limb, has received new support from the result of a recent theoretical investigation by Schwarzschild.[3] So we may safely suppose the beams of average sunlight to come from regions where the mean density of the gaseous medium exceeds that of the atmosphere at the earth's surface. Variations of density amply sufficient to account for the degree of ray-curving assumed by our theory are sure to occur on the paths of those beams. Indeed, nobody will deny that there is motion in the sun. Even if the relative velocities of different parts of the photospheric gaseous material never exceeded, say, 0.5 km per second (an estimate which astronomers, accustomed to velocities

[1] W. S. Adams and C. G. Burwell, *Astrophysical Journal*, **41**, 140, 1915; *Mt. Wilson Contr.*, No. 95, p. 25, 1915.

[2] *Astrophysical Journal*, **38**, 129, 1913.

[3] "Über Diffusion und Absorption in der Sonnenatmosphäre," *Sitzungsberichte der kgl. Akademie der Wissenschaften*, Berlin, 1914.

of hundreds of kilometers, will consider to be very low)[1] such movements would be ten times as fast as terrestrial storms. They would necessarily imply pressure-gradients and local variations of density much greater than those with which our meteorologists are concerned. Besides, vertical convection currents bring into the mixture local differences of temperature and composition, which co-operate with the pressure differences in producing irregular gradients of optical density.[2] One cannot escape the conclusion that the non-existence of such irregular gradients in a medium with internal movements as assumed, would be simply a miracle, and that the idea of it even becomes an absurdity from the point of view of those who assume a state of radial circulation of vapors with varying velocities of several kilometers per second to be a permanent feature of the sun's visible layers, and who, moreover, believe in the frequent occurrence of outbursts of matter at a rate of hundreds of kilometers per second.

If it be once for all admitted that the irregular gradients required by our theory must exist, the rôle of refraction and anomalous dispersion in the production of the fundamental solar phenomena can no longer be disregarded.

A necessary consequence of our hypothesis that the distribution of light in the solar image must everywhere be influenced sensibly by refraction is that we are compelled to examine *every* solar phenomenon from this point of view first. In so doing we cannot be said to

[1] The upper limit of 0.5 km per second was chosen because, according to Hansky (*Mitteil. Pulkowo*, **3**, No. 25, 1908), the maximum velocity of proper motion of sun-spots is 0.4, the mean velocity only 0.15 km per second. If spots are vortices, their movements are real displacements of matter. The velocity of the material circulating round the vortices must be of the same order of magnitude. Hansky has also determined the velocities of granulations. These vary between the narrow limits 2.8 and 4.8 km. In this case, however, there is no sufficient ground for assuming that *matter* is moving at the observed rate. We suppose, in the line of our theory, that we are here witnessing the propagation of condensations and dilatations, like sound-waves. The velocity of such waves in a mixture of hydrogen, helium, and traces of other gases, at 6000°, would exactly fall between the limits mentioned.

[2] We know, from mirage effects, that even the vertical gradient of optical density in our atmosphere may be reversed by a quite reasonable temperature-gradient. Ray-curving, of the order of magnitude occurring in mirage, would produce very marked refraction effects in the sun, where the paths of the beams through the gaseous medium are so much longer, and, therefore, the probable deviations greater.

exaggerate the applicability of the principle of anomalous dispersion. If no additional hypotheses or artificial conceptions are introduced, there is no harm in our inquiring into the logical consequences of also considering anomalous dispersion pure and simple; and if then a solar phenomenon cannot be satisfactorily explained as due only to refraction and scattering there are fortunately several other physical causes at hand which we may try. As a matter of fact, the possibility that other influences such as selective emission and absorption, motion, pressure, magnetic and electric fields, temperature, radioactivity, etc., co-operate in producing the phenomena is by no means denied by the anomalous dispersion theory, and is indeed perfectly compatible with it.

Meanwhile, the remarkable result has to be noticed, that in many cases it is scarcely necessary, at least in first approximation, to have recourse to those other principles, because the main features of a very great number of solar phenomena follow directly from the dispersion theory if we only realize the laws of propagation of light through a rather calmly circulating, extensive mixture of gases. A greater uniformity in our conception of solar phenomena than older theories were able to give is thus attained. The resultant mental image of the sun is, of course, far from complete, but it is capable of extension and improvement by the application of other physical laws.

This review of principles and claims of the anomalous dispersion theory of solar phenomena will perhaps satisfy Adams and Burwell, who felt it as a somewhat disqualifying peculiarity of that theory that its advocates "admit the existence of essentially all of the phenomena in the sun which are required by those who use the more usual explanation,"[1] a fact which, in their opinion, would render the theory rather superfluous. Qualitatively speaking, the quoted statement is quite right, but cannot be said to discredit the introduction of a new, well-established physical principle into astrophysics, nor to make its application superfluous. From a quantitative point of view, however, the statement is incorrect; for the adherents of the dispersion theory can dispense with the

[1] Adams and Burwell, *Astrophysical Journal*, **41**, 143, 1915; *Mt. Wilson Contr.*, No. 95, p. 28, 1915.

enormous velocities of pell-mell motion of solar matter, and with certain artificial hypotheses, and conclusions improbable from a physical point of view, that are constantly puzzling many students of solar physics.

DISCUSSIONS OF ARGUMENTS

1. *Mutual influence of Fraunhofer lines.*—In December 1913 it occurred to me that the dispersion theory required the existence of a definite mutual influence of Fraunhofer lines separated from each other by very short distances, irrespective of the elements to which the lines are due.[1] As the current interpretation of the solar spectrum, which is based on the laws of emission and absorption, and extended by applying the Doppler effect, the Zeeman effect, and the pressure-effect, has never suggested the possibility of a similar interaction, nor seemed to be able to account for it, the actual discovery of the presumed phenomenon would furnish almost conclusive evidence in favor of the hypothesis that the darkness of Fraunhofer lines is mainly an effect of anomalous dispersion. The theory, however, could as yet only predict the general character of the phenomenon, but not the average magnitude of the expected displacements due to mutual influence.

It was hoped that suitable data for testing the foregoing deduction from the theory would be found in St. John's measurements of the Evershed effect in sun-spots.[2] Among the 506 lines of St. John's table, I found 82 members of close pairs so configurated and situated that a definite influence of their other members could reasonably be expected on the basis of the theory. The selection of the pairs and the determination of "normal" or "standard" displacements was a delicate process; I believed that I had applied it fairly. The result seemed to be in accord with the theory, and I submitted the matter with confidence to the judgment of other investigators.[3]

A few months later St. John published the elaborate article[4] in which he criticized my treatment of his observations very severely,

[1] Provided those elements coexist in the mixture at the same levels.

[2] *Astrophysical Journal*, **37**, 322, 1913; *Mt. Wilson Contr.*, No. 69.

[3] *Astrophysical Journal*, **40**, 1, 1914.

[4] *Ibid.*, **41**, 28, 1915; *Mt. Wilson Contr.*, No. 93.

in several respects justly. I had not overcome the difficulties of handling rightly the Mount Wilson data, nor had I entirely avoided bias. St. John made certain alterations in the method of grouping and comparing the measured displacements, added a number of omitted and of new cases, and thus reached the conclusion that there was no indication at all of a mutual influence.

Although I am satisfied by St. John's improved discussion of the data that, in the Evershed effect, mutual influence is not so conspicuous as my defective treatment of those measurements had made it appear, I still believe that future research will bring it to light. This conviction has recently received strong support from S. Albrecht's discovery that *in the general solar spectrum* those iron lines which have close companions are indeed systematically displaced, in perfect harmony with the requirements of the anomalous dispersion theory.[1]

For a mean separation of the interacting lines of 0.22 A the average shift was 0.007 A, if toward the violet; and 0.005 A, if toward the red.

There is no contradiction between St. John's failure as yet to find mutual influence in the Evershed effect, and Albrecht's success in disclosing the predicted phenomenon in the general solar spectrum; for we can easily show that in the latter case the effect should be sensibly greater than in the former.

To this end we must reproduce the diagram (Fig. 1) and its explanation, by which in the *Astrophysical Journal*, 40, 12, 1914, a possible mutual influence of Fraunhofer lines was illustrated.

The value n_0 which the refractive index would have in the part of the spectrum under observation, if this were free from absorption lines, is here supposed to be >1. The effect of a line B is to reduce the indices on its violet side and to raise them on its red side, as indicated by the partly broken curves. The line A, if isolated, would produce its own anomaly in the dispersion-curve as shown in A_1. If A were situated near B, in one of the positions A_2 or A_3, that anomaly would have a somewhat different form in consequence of its being superposed upon one of the branches of the dispersion-curve due to B. The refracting properties of the

[1] *Astrophysical Journal* 41, 333, 1915.

medium, being determined by the value of $n-1$, will be different in the three cases represented by A_1, A_2, and A_3.

Only those waves for which the absolute values of $\pm(n-1)$ exceed a certain minimum value will contribute sensibly to the formation of the Fraunhofer line, and will follow sufficiently curved paths in the outer parts of a vortex region (a sun-spot) to give rise to recognizable effects of refraction in the spectrum of the

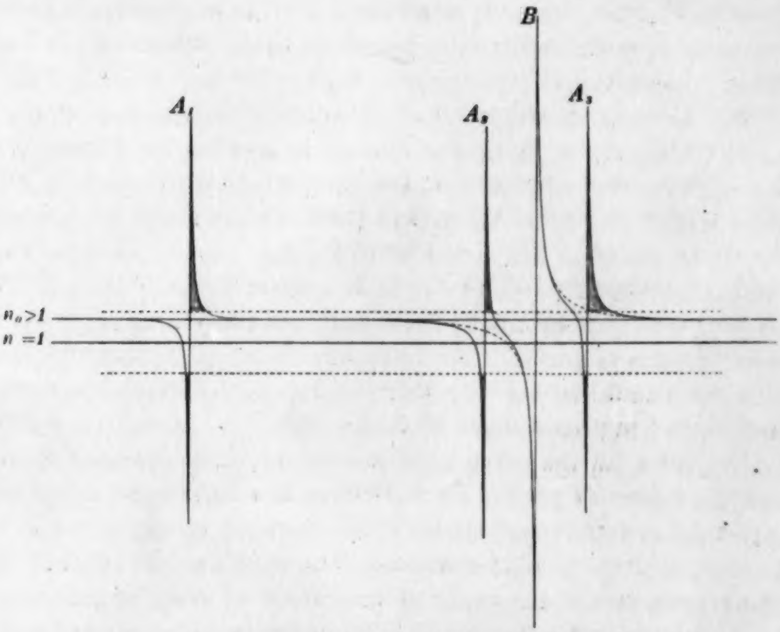


FIG. 1.—Mutual influence of Fraunhofer lines

penumbra. This is indicated in the figure by means of the two broken lines drawn at equal distances above and below the line $n=1$. We may assume that only the parts of the dispersion-curve lying outside the zone between these broken lines are material to the formation of the dispersion bands enveloping the absorption lines. The R-light corresponding to the shaded area above the zone is responsible for the displacements toward the red observed at the peripheral edge of the penumbra; the V-light corresponding to the shaded area below the zone causes the displacements toward the violet at the central edge.

Now comparing for the lines A_1, A_2, and A_3 the horizontal distances between the "centers of gravity"[1] of their R-area and V-area, we at once realize that A_2 will show a smaller "relative displacement" (distance) than A_1, and A_3 a greater relative displacement than A_1; but it is also evident from the figure that the difference between the cases A_3 and A_1 is not so marked as that between A_2 and A_1.

Such is the anticipated manifestation of mutual influence if we are considering the relative displacements involved in the Evershed effect. We refer to it as case α.

In the general spectrum of the disk, with the exception of spots, the R-area and V-area co-operate in forming the Fraunhofer line by irregular refraction and scattering. With respect to the uninfluenced condition A_1, A_2 will there appear displaced toward the violet, and A_3 a little less toward the red. This is the effect as studied by Albrecht. We refer to it as case β.

Now it is clear that both the R-area and the V-area of A_2 have their "centers of gravity" displaced toward the violet as compared with the condition A_1; the *mean* of the two shifts is what we observe as "mutual influence" in case β.

In case α, on the other hand, we observe only the finer detail that the center of gravity of the R-area is a little more displaced than that of the V-area; it is the *difference* of these shifts which determines the reducing influence of B on the Evershed effect of A.

Since in case β the order of magnitude of the displacements due to mutual influence proved to be 0.006 A, the mean value in case α could scarcely be expected to exceed 0.002 A, even if the separate residuals were weighted, as proposed by Albrecht, and if lines suspected of being sensibly influenced by companions on *both* sides had been omitted from the list.

This would explain why St. John obtained a negative result. As far as the available measurements of the Evershed effect were concerned, St. John's statement that the presumed mutual influence was well suited for making a "quantitative" and definitive test

[1] The term "center of gravity" is used for the sake of convenience. Properly speaking, the estimated location of the lines will of course depend on considerations somewhat different from those involved in the determination of the center of gravity of the said area.

of the theory was premature. A positive result would have furnished a *qualitative* confirmation; the negative result left the problem in suspense; but since the existence of mutual influence has been definitely established by Albrecht, we have no doubt that it will be detected in the Evershed effect too.[1]

Perhaps it is not superfluous to remark that the absolute magnitude of the displacements due to mutual influence (say 0.006 A) should not be thought to represent an approximate measure of the total share which anomalous dispersion has in the phenomena of the Fraunhofer spectrum. That share must be many times greater. Mutual influence is but a peculiar *differential* effect by which the *integral* effect (i.e., almost the total width of the Fraunhofer line) reveals its nature. Only if the Fraunhofer lines are *mainly* due to anomalous dispersion will they be able to show a mutual influence of the observed kind and magnitude.

Another form of mutual influence, a cumulative effect, is the fluctuation of n_0 along the spectrum. Many of the small irregular differences in wave-length between lines of the solar spectrum and corresponding lines observed in the laboratory will probably find their explanation in such consequences of anomalous dispersion.

Before closing this section we draw attention to the rather large number of cases which seem to be exceptions to the rule required by the theory. For companion toward the red Albrecht found that 11 per cent of the lines gave values of opposite sign from the mean, while for companion toward the violet as many as 26 per cent showed opposite sign. These need not be real exceptions. Indeed, n_0 fluctuates, and in some regions of the spectrum might be <1, which would reverse the sign of the required effect. Besides, small uncertainties in the wave-lengths of the terrestrial comparison lines are not excluded.

[1] Differential shifts of this kind are by no means proportional to the intensity of the influencing lines. A reasonable analysis of the various possible cases of line-shift, which diagrams of the type of Fig. 1 could illustrate, would easily bring out the error in St. John's presumption (*Mt. Wilson Contr.*, No. 93, p. 21) that the great wings of such very strong lines as H and K should be precisely the place for mutual influence (in the Evershed effect) to manifest itself. As a full treatment of this question would require more space than can here be spent on it, I leave it to the reader, and will only remark that in such large wings the slope of n_0 is rather slight.

2. *Anomalous dispersion in the laboratory and in the sun.*—Several of the objections raised by St. John in his criticisms are founded on his opinion that astrophysical deductions from the dispersion theory have been made to the neglect of, and in contradiction to, the results of laboratory work on anomalous dispersion. He says:

It is worthy of note that Professor Julius takes no account of laboratory results when considering center and limb displacements; but to prove that very strong and very weak anomalous dispersion make the displacements small and that intermediate values give large displacements, he classifies the lines simply according to line intensity, irrespective of their known anomalous dispersion. This appears to assume that anomalous dispersion is proportional to line intensity. Likewise, there is no attempt to correlate the effects of mutual influence with what is known of the power of the influencing lines to produce anomalous dispersion phenomena. It seems again to be assumed that all lines of the same intensity are equally effective.[1]

This is all nearly true; but what St. John here represents as a procedure to be censured and an additional assumption without foundation is nothing but a paraphrase of my fundamental hypothesis that Fraunhofer lines are, in the main, dispersion bands. This hypothesis implies, of course, that in first approximation the intensity of the lines should be proportional to the anomalous dispersion—in the sun—of adjacent waves.

I think it more reasonable to try the supposition that the intensity and further properties of the Fraunhofer lines are intimately connected with the amount of anomalous dispersion effects *in the sun*, than to assert that, if dependent at all upon anomalous dispersion, they should be proportional to such effects of anomalous refraction as one has hitherto succeeded in producing *in the laboratory* under circumstances not comparable with those obtaining in the sun.

A quotation made by St. John[2] from the earliest paper ever published on the subject (1900) shows that the simple but superficial idea of expecting some close agreement between the intensity of chromospheric lines and the amount of anomalous dispersion manifested by the corresponding lines in the laboratory was also the

[1] *Astrophysical Journal*, **41**, 55, 1915; *Mt. Wilson Contr.*, No. 93, p. 28.

[2] *Astrophysical Journal*, **41**, 51, 1915; *Mt. Wilson Contr.*, No. 93.

first to occur to me; but I soon perceived that the connection between solar phenomena and this kind of laboratory results could not possibly have the simple form which St. John still continues to demand.

The present theory of light requires that the refracting power $\pm(n-1)$ of a medium varies rapidly for waves in the neighborhood of each of its proper periods. Anomalous dispersion may be called a general property of matter. Such expressions as "the power of the lines to produce anomalous dispersion effects,"[1] though convenient and often used, appear to be misleading. It is not the lines that produce the effects; it is the medium, and only if circumstances are favorable.

Suppose we have a homogeneous gas giving a monochromatic absorption line. Let a parallel beam of white light traverse a layer of it one meter thick and of constant density throughout: the line will show very thin and sharp (or perhaps will not show at all). Let the thickness of the layer increase to several kilometers: anomalous scattering will envelop the line in narrow, tiny wings. Let the gas become disturbed: anomalous refraction in the irregular density-gradients will widen and darken the wings.[2]

Thus, temperature, mean pressure, average density, electric and magnetic conditions remaining the same, a line may show very different aspects according as the medium is more or less extensive and disturbed.

Great extension and optically effective disturbances of density of gaseous media are, as a rule, hard to realize in the laboratory, but are sure to govern solar phenomena.

The success of a laboratory experiment on anomalous dispersion depends upon the possibility of obtaining the gas in such a quantity and condition that it presents a sufficiently great and controllable density-gradient to make the rays deviate appreciably; or, if studied by the method of the interferometer, that a sufficient number of electrons of the considered proper period are present to

[1] *Ibid.*, p. 56.

[2] Phenomena of this kind might, for example, be studied by observing how the atmospheric lines of the solar spectrum vary with the sun's altitude and the conditions of our atmosphere.

affect the velocity of light appreciably. A negative result of the experiment does not signify that "the line in question lacks the power of producing anomalous dispersion effects," but simply that, in the medium, the external conditions necessary for their appearance were not fulfilled.

Effects of anomalous refraction are much more easily obtainable with sodium than with magnesium or iron. In the solar spectrum several lines of magnesium and iron are nevertheless more conspicuous than the sodium lines. This, however, does not at all conflict with the anomalous dispersion theory of solar phenomena, for it is quite possible, for instance, that the visible layers of the sun contain much less sodium than iron and magnesium.

The foregoing remarks may suffice to refute the objections based on the absence of proportionality between properties of Fraunhofer lines and amount of dispersion effects hitherto obtained in the laboratory.

3. *General shifting of the Fraunhofer lines to the red.*—Two classes of these general displacements are usually distinguished: the sun-arc (or rather center-arc) shifts and the limb-center shifts.

The interpretation of the center-arc shifts on the basis of the Doppler principle would imply that the various constituents of the solar atmosphere are descending with specific velocities all over the solar surface. Line displacement due to this cause should, of course, decrease toward the limb. The observations, however, show an increase toward the limb; the limb-center shifts had therefore to be considered as a separate phenomenon, superposed upon the center-arc shifts. Evershed again invokes the Doppler principle for explaining them, and thus has to admit the existence of a selectively acting repulsive force exerted by the earth on the solar gases.

Another interpretation often applied to both classes of shifts is based on the pressure-effect. The limb-center shifts would on this view indicate an increasing preponderance of the pressure-influence due to the lower levels with respect to the pressure-influence due to the higher levels as we proceed from the center toward the limb. The interpretation requires that there be some parallelism between the relative displacements of the lines of a given element in the

solar spectrum and those of the corresponding lines under pressure in the laboratory.[1] The fact that the agreement is far from satisfactory induced Evershed to reject the pressure theory, and to have recourse to the hypothetical repulsing force mentioned above.

According to the dispersion theory Fraunhofer lines are, as a rule, unsymmetrical; this would be the cause of their apparent displacements. Width, intensity, and degree of asymmetry will vary with the absolute quantity, the distribution of density, and the further conditions of the gases traversed by the light which comes from the interior of the sun. Quantity and condition of traversed material vary gradually from center toward limb. This produces the difference between limb lines and center lines, which lines, for the rest, must have a very similar origin. It would therefore appear safer to draw conclusions from a comparison of limb lines with center lines than from a comparison of center lines with arc lines, for in the latter case many unknown differences of condition might be involved.

It is a fact that the only broad regularity until now observed in the limb-center shifts is a dependence on line intensity of just the kind predicted from the point of view of the anomalous dispersion theory. Other interpretations of the displacements had not suggested the possible existence of a connection between the general limb-center shift and the intensity of the lines.

St. John objects[2] that the center-arc shifts appear not to be subject to the same law. I question whether there are sufficient grounds for this contention as yet. From all the data hitherto obtained in the matter by Evershed and Royds, Fabry and Buisson, and Schwarzschild we derive a result different from that given by St. John, although, it must be granted, not in *clear* agreement with the law. Further evidence must be awaited, especially because the wave-lengths of many arc lines prove to be dependent on certain still uncontrollable circumstances, which seem to make center-arc shifts less reliable than limb-center shifts.

[1] There is more reason to demand such parallelism between solar and laboratory results when solar line displacements are to be ascribed to pressure-effects than when anomalous dispersion effects are called in for explanation, because pressure-shifts are not dependent on the *absolute* quantity of matter under observation.

[2] *Astrophysical Journal*, 41, 61, 1915; *Mt. Wilson Contr.*, No. 93, p. 34.

A few lines of the solar spectrum are shifted to the violet instead of to the red. St. John supposes these cases to be unaccounted for by the anomalous dispersion theory; but he forgets that n_0 may occasionally be < 1, which would reverse the direction of the asymmetry of the Fraunhofer line.

The last paragraph of the section devoted by St. John to general displacement[1] should be appreciated as a friendly turn toward Mr. Adams, in which I would gladly join, since I have neither said nor thought that Mr. Adams considered pressure as the *only* effective agent concerned in limb-center shifts. Meanwhile it remains true that Adams—quite naturally from his point of view—has not foreseen the existence of a rather simple connection between displacements and line intensity.

4. *The flash spectrum and anomalous dispersion.*—The darkness of Fraunhofer lines and the brightness of flash lines are from the point of view of the anomalous dispersion theory complementary phenomena—two aspects of one and the same process. They are both caused by selective scattering of the waves whose periods differ very little from the proper periods of the medium. It is convenient to distinguish molecular scattering or diffusion from the coarser scattering due to refraction by the irregularities of optical density of the medium, although the two kinds perhaps pass gradually into each other (opalescence). Diffusion, besides depending on the refractive index, is inversely proportional to the fourth power of the wave-length, whereas refractional scattering varies only with n.

If there be a bright background, scattering will diminish its brightness. This explains the Fraunhofer lines. The scattered light itself, if seen against a dark background, constitutes the lines of the flash spectrum.

Suppose we direct a powerful telescope on a point of the sun's edge. One half of our small field of view will be a bright, the other half a dark, background. Direct light coming from the limb would have to travel a long way through the gaseous medium (20,000 km through a layer 300 km thick, 40,000 km through a layer 1200 km thick).

[1] *Op. cit.*, pp. 64 or 37.

From a discussion of the effects of scattering in the long, narrow column of solar gases through which the light from the limbs has to pass, we should be able to deduce the principal properties of both the flash spectrum and the Fraunhofer spectrum of the limb. The task is not very simple but is, I think, quite definite; every single case is capable of being considered from this point of view, and the *general* features of those two types of spectra will be seen to follow directly from the proposed mental image.

It should first be recalled that the localization of the sun's edge depends, according to our interpretation of the photosphere,[1] upon refraction in the irregular density-gradients. The diameter of the photospheric disk is determined by the average refractional scattering of the waves not appreciably subject to anomalous dispersion. Waves suffering sensible anomalous refraction in layers near the level of the apparent limb will produce somewhat larger solar disks of their own.

To every value of $\pm(n-1)$ corresponds a definite magnitude of disk. The ring-shaped parts of those larger disks which protrude beyond the principal disk form together the chromosphere.[2]

If the slit of a spectrograph be set exactly tangent to the principal disk, it cuts the protruding rings, whose light will furnish the flash spectrum, every line of which consists of waves on both sides of the proper periods of the medium. On a flash line the corresponding Fraunhofer line may be visible ("double reversal") because, properly speaking, the slit was still *on* the disks which are formed by the waves closely approximating the proper period in question. The visibility of this double reversal requires, of course, a very accurate setting of the slit; so we at once understand why Adams and Burwell

[1] *Astrophysical Journal*, **38**, 129, 1913.

[2] In first approximation we neglect the radial density-gradient in comparison with the irregular gradients. Refraction in the small-scale density-gradients contributes to both the R-light and the V-light of the chromospheric lines. By the term "small-scale gradients" I refer to a condition of differing density in spaces so small that they cannot separately be seen at the distance of the earth. The objection made by Adams and Burwell at the top of p. 26 of *Mt. Wilson Contr.*, No. 95, viz., that the wave-lengths of the chromospheric bright lines do not show any marked systematic variations, will thus be met. We are dealing with an average effect. Large-scale density-gradients in higher levels manifest themselves as prominences; in their spectrum it is often observed that R-light and V-light are locally separated.

found that "double reversal seems to be a universal characteristic of all lines in the chromospheric spectrum at a low level,"[1] whereas in eclipse spectra obtained with the prism camera double reversals are restricted to the stronger lines.[2]

Important results of recent observations of the flash spectrum (by Hale and Adams, Mitchell, Adams and Burwell) are:

1. There is a regular progression of the lengths of flash spectrum arcs with intensities of the Fraunhofer lines.[3]

2. The difference in intensity between flash lines and Fraunhofer lines is positive for very weak solar lines, diminishes gradually as stronger lines are considered, passes through zero, and attains increasing negative values[4] for solar lines up to intensity 6 (which law might also be expressed thus: flash line intensity increases with solar line intensity, but at a slower rate).

3. Enhanced lines are of an exceptionally great intensity in the flash spectrum, and nearly always appear as double reversals. The same class of lines generally are weakened and almost deprived of their wings in the spectrum of the disk close to the limb.

4. According to Adams and Burwell flash lines are displaced to the red relative to the Fraunhofer lines at the center of the sun by an average amount of about 0.006 A. For the mean of the limb-center shifts the same observers found 0.008 A, but they do not feel justified in considering the slight difference between those two mean values as certainly real.[5] Earlier measurements by Hale and Adams had given an equal small difference of opposite sign. So we may in first approximation admit that flash lines are not displaced appreciably with respect to Fraunhofer lines of the limb spectrum.

[1] *Astrophysical Journal*, **41**, 136, 1915; *Mt. Wilson Contr.*, No. 95, p. 21.

[2] The fact that the general duplicity of chromospheric lines, which was predicted in the earliest paper on anomalous dispersion in the sun (1900), did not clearly appear on the best eclipse plates has been adduced as an argument against the interpretation of the chromosphere on the basis of the dispersion theory. Now that duplicity seems after all to be a universal property of those lines, the subject might perhaps again serve as an argument, but now in favor of the theory.

[3] St. John, *Astrophysical Journal*, **40**, 358, 1914; *Mt. Wilson Contr.*, No. 88, p. 3.

[4] *Astrophysical Journal*, **40**, 368, 1914; *Mt. Wilson Contr.*, No. 88, p. 13.

[5] *Astrophysical Journal*, **41**, 143, 1915; *Mt. Wilson Contr.*, No. 95, p. 28.

These four points will now be briefly discussed from the point of view of our theory.

1. We refer to the concept, mentioned above,[1] of the long column of gases through which a small field of view surrounding a point of the limb is observed. To every line there corresponds, within the column, a certain number of carriers of a given definite proper period. An immense stream of light goes across the full length of the column. The amount of selective scattering (molecular and refractional) will depend upon that number of carriers. Evidently there must be a close relation between the intensity of the scattered light which in the dark half of the field of view produces the flash line, and the loss by scattering which in the bright half causes the darkness of the Fraunhofer dispersion line. A regular progression of flash intensity with Fraunhofer intensity is thus easily accounted for. At the same time, increasing intensity of the flash lines will make the flash-arcs, obtained with the prism camera, show longer; and as the distance from the sun's edge at which scattered light is still visible will also increase with the number of carriers in our column, we have an additional cause for the arcs of the flash spectrum to increase in length with the intensity of the solar lines.

This is the place to notice a difference of some importance between the conclusions to which the current and the new interpretation of flash-arcs lead.

If the flash-arcs be considered as due to selective emission, the length of an arc would furnish us with an indication of the height to which the element in question generally rises in the solar atmosphere. Elements with high atomic weights show, as a rule, short arcs, and it seems reasonable that they should be found only at low levels. But carbon arcs being equally short, a low level has also to be assigned to carbon. There are other discrepancies, or at least difficulties, of this kind. Different lines of one and the same element (Ca, Sr, Ti, Fe, etc.) often indicate, on this view, very different levels, a fact requiring for its explanation the additional hypothesis that the emissive properties of such elements vary with height.

[1] Cf. p. 58.

If, on the other hand, we explain the flash light as due to scattering, the lengths of the arcs (depending on the distances outside the limb at which scattered light is still visible) are determined by the concentration with which each kind of carriers of proper periods is represented in the mixture. It is not denied that the composition of the mixture will probably change with height; but *even* if it remained the same at all heights, carriers weak in number would give short flash-arcs, carriers strong in number would produce long arcs. On this view, the short flash lines of carbon do not indicate low level of that element, but feeble concentration; and as the different carriers corresponding to the various lines of an element need not be equally numerous nor equally effective, there is no difficulty in understanding why an element should give flash lines of very different lengths.

2. The peculiarity mentioned in point 2 is very interesting, but it seems hazardous to derive any positive conclusion from it as yet, because the scale of flash intensities is not easily comparable with the scale of intensities assigned to Fraunhofer lines. I would only venture to suggest that, according to the anomalous dispersion theory, the relatively great flash intensity which corresponds to the weakest Fraunhofer lines may find its explanation in the conception that flash light is always a part of the strong photospheric light, however scanty the scattering carriers may be, and that a weak light on a dark background is more conspicuous and will also give stronger contrast on the photographic plate than a slight obscuration on a bright background.

3. The following hypothesis would seem useful for elucidating the special behavior of the *enhanced lines* in the spectra of flash and limb by means of the notions derived from our theory.

We simply suppose those solar lines to be due to carriers whose percentages in the gaseous mixture are relatively great.

All scattering effects produced by such carriers will then be very strong. The "solar disks" peculiar to their various R-light and V-light waves are much greater than the principal solar disk (a fact which, in the language of the current theories, would be expressed by saying that the corresponding elements occupy a thick layer in the chromosphere). From this consideration it

follows: (*a*) that, if the slit of a spectrograph be set tangent to the principal disk or at a small distance outside, those chromospheric lines will appear very bright and long, and nearly always as double reversals; (*b*) that in eclipse photographs taken with the prism camera the arcs will be long and broad; (*c*) that, if the slit of a spectrograph be set *on* the disk at a certain short distance from the limb, it will be at *greater* distances from the edges of the peculiar disks which correspond to the strongly refracted waves environing the proper periods of the carriers. This involves weakening of the dark wings in the limb spectrum. For in each monochromatic disk there is a rapid increase of brightness from the edge inward, so that at the selected place of the slit, large-disk waves will have gained more in intensity than average-disk waves, and therefore will show relatively bright. Large-disk waves are those corresponding to the wings; they will be brighter, as compared with the rest of the spectrum, than they are in the spectrum of the central parts of the disk. It is true that for all kinds of light the *absolute* brightness decreases from center toward limb; but for the strongly scattered and refracted light the region of rapid decrease is shifted a little outward; consequently such light is less weakened, at a given point near the limb, than the average light. Its relatively greater strength in the spectrum of the limb manifests itself as a narrowing of the line or a weakening of its wings.

There is an additional cause, dependent on refractional scattering, by which lines, due to principal constituents, will have their wings gradually weakened toward the limb.

For comparison we first consider the rôle of molecular scattering. According to Rayleigh's well known formula the coefficient of molecular scattering increases with decreasing wave-length, and also with decreasing distance from each proper frequency of the medium. The "fogginess" of the sun therefore varies very strongly with the kind of light used in the observations; it is greater with violet than with red light, and greater with R-light and V-light than with waves of adjacent blank parts of the spectrum. Fogginess hinders us from discerning structural particulars at lower levels (hence the varying character of spectroheliograms as the selected wave-length approaches the center of a line), and causes

weakening of the transmitted light (hence, in substance, the darkness of the Fraunhofer lines).

Now, besides molecular scattering we have scattering by refraction in irregular gradients, which is especially strong with the R-light and V-light of lines of principal constituents. By this kind of scattering the peripheral parts of the disk will appear illuminated at the expense of the central parts (compare the case of a globe of frosted glass inclosing a source of light); the result is a relative weakening and narrowing of the winged lines in the limb spectrum.

4. The rather close agreement of the wave-lengths of flash lines with the wave-lengths of the Fraunhofer lines at the limb is in perfect harmony with the requirements of the anomalous dispersion theory. Indeed, both classes of lines originate from the same scattering process in almost the same long column of gases. Any small increase in the degree of scattering would, as a rule, make the Fraunhofer line show darker, the flash line brighter. If, therefore, scattering effects be somewhat greater for the R-light than for the V-light, this will manifest itself as a displacement of the flash line as well as of the Fraunhofer line toward the red, by nearly the same amount. It is not[1] difficult to understand, from this point of view, why double reversals should nearly always be symmetrical, although the refracting power of the solar atmosphere is, as a rule, greater on the red side of an absorption line.

5. *Supplemental remarks.*—A few of the objections made by St. John are left which have not been given explicit or implicit consideration in the preceding pages.

On p. 39 of *Mt. Wilson Contr.*, No. 93, St. John refers to observations showing displacements when the slit of the spectrograph is perpendicular to the radius of the solar disk passing through the center of the umbra of a spot. From earlier publications by Evershed and St. John, I understood that such displacements were rather exceptional and not systematic; so I suggested that they

[1] Cf. Adams and Burwell, *Mt. Wilson Contr.*, No. 95, pp. 26–28, and *Communications to the Nat. Acad. of Sciences*, No. 4. It will be noticed that our argument leads to conclusions directly opposite to those which Adams and Burwell deduced from the anomalous dispersion theory as they apprehended it.

might be accounted for by admitting unequal refraction at opposite edges of the spot. Since observations by more refined methods have shown that such displacements "occur and persist for weeks in regular and symmetrical spots," I am inclined with St. John to ascribe them in part to the Doppler effect connected with vortex motion. The rotary motion round a vortex is more likely to be sufficiently rapid to be observed by its Doppler effect than the radial movement in the spot; and Hansky's above-mentioned observations (cf. p. 47, n. 1) on proper motions of sun-spots seem to prove that real currents of matter at the rate of 0.1 to 0.4 km per second may occur in the sun.

On p. 40, *op. cit.*, St. John again emphasizes the fact that in the spectra of the penumbrae some strong lines show relative displacements in a sense opposite to the direction of the general Evershed effect, and says that up to the present the anomalous dispersion theory has been unable to suggest an explanation. I have twice drawn attention to this point myself (on p. 25 and p. 30 of the paper criticized)[1]; I suggested the cause of the difficulty of the question, and reserved the subject for further investigation. So I do now, since a satisfactory solution has not yet been found.

A third point is that the dispersion theory has until now furnished no explanation of the differences characteristic of the elements such as those shown by the displacements of the lines of iron, titanium, and the heavy elements when lines of the same region and intensity are compared. Thus far no special attention has really been given to this important subject, because, from our point of view, it seemed necessary first to investigate general optical solar phenomena irrespective of the separate elements concerned. But in future the matter should of course also be considered. For the present I would only suggest that the varying composition of the gaseous mixture with increasing height will probably furnish the key of the problem.

We owe to the rigorous critique exercised by St. John and Adams and Burwell on some of my recent papers the fact that

[1] *Astrophysical Journal*, 40, 1, 1914.

attention has been called to various deductions from the anomalous dispersion theory which required ampler discussion than had been given them up to the present, and also that some cases of misapprehension regarding the claims of the theory have come to light. These weak points could now be corrected or explained. I see no reason, however, for modifying the principal conclusions arrived at in those former papers, except in so far as clear evidence of mutual influence of Fraunhofer lines has not yet been obtained from observations on the Evershed effect in sun-spots, but seems to be available in the results of Albrecht's investigations on the wave-lengths of iron lines in the solar spectrum.

UTRECHT
September 1915

THE ULTRA-VIOLET SPECTRUM OF KRYPTON

By E. P. LEWIS

The most complete investigation of the spectrum of krypton has been made by Baly,[1] but he did not carry his measurements in the ultra-violet beyond λ 2418. Recent photographs, taken by the writer, of the spectra of rare gases in vacuum tubes prepared by Hilger showed that the spectra of both krypton and xenon extend to a great distance in the ultra-violet. The intensity of the lines of both these gases diminishes rapidly in approaching λ 2100 to the same degree as the spectra of zinc, cadmium, and other metals known to contain strong lines in this region. The inference is that the apparent termination of their spectra at this point is due to the absorption of the thick quartz system, and that the spectra actually extend into the Schumann region.

The spectrograph used was the large two-prism instrument previously described.[2] Between the limits λ 2100 and λ 2400 the dispersion lies between 1.5 and 3 units per millimeter—a dispersion considerably greater than that in the first order of the 15-foot grating in this laboratory. The light-power is such that three minutes was a sufficient exposure with condenser and spark-gap and ten minutes with self-induction added.

The wave-lengths of the new krypton lines are given in Table I. Some comparisons with Baly's results indicate the probable accuracy of the measurements. Beyond λ 2200 the results are not so reliable, on account of the limited number of standard lines in this region. The wave-lengths were calculated from Hartmann's interpolation formula, corrected by comparison with iron and cadmium lines.

Comparison of the spectra of helium, neon, krypton, and xenon indicated that all these gases were nearly free from contamination with each other. Helium has no lines and neon but few in the region investigated; and but few lines were observed which may

[1] *Phil. Trans.*, A **202**, 183, 1903.

[2] *Astrophysical Journal*, **23**, 390, 1906.

TABLE I

λ	C.	S.-I.	λ	C.	S.-I.	λ	C.	S.-I.
2145.9	1	......	2265.70	1	......	2341.97	1	......
2149.3	1	......	2266.25	1	......	2343.11	3	......
2155.6	1	......	2267.03	1	......	2344.55	8	5
2159.5	1	......	2268.06	1	......	2345.62	6	......
2162.7	2	......	2270.59	2	......	2348.27	1	......
2164.6	2	......	2271.90	1	......	2350.08	1	......
2166.1	1	......	2272.68	1	......	2353.12	2	......
2169.2	1	......	2273.15	6	2	2353.95	10	9
2170.8	2	......	2273.63	3	......	2355.65	1	......
2172.2	2	......	2274.70	3	......	2356.60	1	......
2173.7	1	......	2277.50	7	......	2358.80	3	......
2177.7	2	......	2279.79	5	......	2360.40	3	......
2179.3	1	......	2280.54	1	......	2362.18	4	......
2185.4	2	......	2281.26	1	......	2363.01	8	......
2186.8	1	......	2282.92	10	3	2363.74	1	......
2188.3	1	......	2284.43	1	......	2365.03	1	......
2192.1	2	......	2285.61	2	......	2365.80	5	......
2193.6	1	......	2287.75	10	3	2366.35	1	......
2197.4	2	......	2289.31	1	......	2368.30	4	......
2206.6	2	......	2290.30	1	......	2369.25	4	......
2211.88	2	......	2291.26	6	......	2370.27	2	......
2213.05	3	......	2292.45	1	......	2371.60	8	......
2215.70	2	......	2296.02	2	......	2372.93	1	......
2216.08	1	......	2297.98	1	......	2373.81	6	......
2216.72	1	......	2299.02	6	......	2375.68	10	6
2217.59	1	......	2300.35	8	......	2376.85	3	......
2218.26	1	......	2301.69	8	......	2382.99	1	......
2219.19	2	......	2302.88	6	......	2383.91	1	......
2221.87	1	......	2304.37	1	......	2384.80	1	......
2225.10	2	......	2304.78	1	......	2385.97	1	......
2227.96	6	2	2305.33	1	......	2387.23	1	......
2230.70	1	......	2307.19	1	......	2388.05	2	......
2232.26	1	......	2309.42	1	......	2389.62	2	......
2232.90	1	......	2311.97	8	......	2390.65	4	......
2233.76	1	......	2314.21	8	3	2392.92	7	3
2234.30	2	......	2315.45	9	4	2394.06	8	......
2237.07	5	......	2316.29	10	4	2394.90	1	......
2243.10	1	......	2317.81	1	......	2397.09	5	......
2244.09	1	......	2319.01	1	......	2398.38	10	......
2244.38	1	......	2319.70	1	......	2399.21	2	......
2245.34	6	......	2320.92	6	......	2400.30	5	......
2247.19	1	......	2322.40	2	......	2401.66	1	......
2248.58	2	......	2323.85	4	......	2402.62	2	......
2250.22	3	......	2324.85	3	......	2403.14	3	......
2251.13	2	......	2326.55	5	......	2403.80	3	......
2251.96	2	......	2328.21	1	......	2406.42	6	......
2252.82	1	......	2329.30	8	......	2407.28	5	......
2253.47	1	......	2330.35	1	......	2408.59	7	3
2254.14	1	......	2331.55	2	......	2409.15	8	3
2255.10	3	......	2332.28	1	......	2412.11	3	......
2259.83	2	......	2333.33	1	......	2413.93	9	3
2260.72	2	......	2336.75	1	......	2415.06	9	3
2262.94	1	......	2339.15	1	......	2416.31	3	......
2263.71	2	......	2340.05	2	......	2416.90	3	......
2264.69	1	......	2340.93	5	......			

possibly be common to the spectra of krypton and xenon. Argon is a possible impurity in the krypton, and some of the observed lines lie very near argon lines given in Eder and Valenta's table; but in most cases the observed intensities make their identity improbable.

The addition of a small inductance to the circuit previously containing spark-gap and condenser caused a marked change in the spectrum. There was a general reduction in intensity, but if the time of exposure was increased until some of the stronger lines in the two spectra were of about the same intensity, it was found that a number of other strong lines were completely suppressed, while a smaller number persisted, but with greatly reduced intensity. The intensities are compared in Table I under the heads C. (condenser and spark-gap) and S.-I. (inductance added).

TABLE II

λ	C.	S.-I.	Baly	λ	C.	S.-I.	Baly
2418.26.....	10	2	2418.13	2446.54.....	8	......	2446.56
2420.31.....	10	2	2420.30	2452.43.....	6	......	2452.38
2425.15.....	5	......	2425.15	2453.39.....	6	......	2453.37
2426.43.....	9	8	2426.46	2454.15.....	4	......	2454.19
2428.43.....	10	9	2428.44	2455.35.....	2	......	2455.42
2439.33.....	8	......	2439.32	2456.18.....	8	4	2456.16
....	......	......	2439.64	2457.78.....	8	......	2457.79
2440.11.....	5	......	....	2459.68.....	7	......	2459.74
2442.67.....	7	......	2442.68	2464.86.....	8	7	2464.87

Tables I and II show that many strong lines are completely suppressed by self-induction, some affected only slightly, and others reduced in intensity in varying degrees. The study of the effect of self-induction was carried into the field of Baly's measurements up to about λ 3700. Beyond this point it did not seem worth while to go, on account of the small dispersion of the spectrograph in this region. In general, it may be said that the suppressive effect of self-induction appeared to be less pronounced in the visible region than in the ultra-violet. Table III gives the lines (additional to those contained in the preceding table) which were only slightly affected in relative intensity by self-induction (all being reduced in approximately the same degree). The interpolation formula was used merely to identify the lines, the wave-lengths being those given by Baly.

The table of new lines shows that a large number are completely suppressed by self-induction. The same is true for the region examined by Baly. In Table IV is given a list of the stronger lines only in the latter region which are so affected. In some cases (indicated by an asterisk) the suppression is not complete, an exceedingly faint trace of the line being seen when self-induction is used.

TABLE III

λ	C.	S.-I.	λ	C.	S.-I.
2503.97	8	6	3446.66	7	6
2506.66	9	6	3460.24	7	6
2589.19	7	5	3470.19	6	5
2592.57	8	8	3488.74	8	7
2620.54	7	6	3503.38	7	5
2643.18	5	4	3535.48	8	7
2648.26	8	7	3544.69	9	9
2712.50	8	8	3589.79	7	6
2733.38	5	4	3600.05	8	8
2742.67	6	5	3608.02	8	8
2795.92	7	6	3623.74	4	3
2816.58	8	8	3632.02	9	9
2833.11	7	7	3637.63	6	5
2967.37	6	5	3654.11	9	9
3151.06	8	8	3669.16	8	8
3200.53	6	5	3680.52	8	8
3207.91	8	8	3686.30	8	8
3405.28	8	8	3718.17	10	9
3427.84	6	5			

Between these extreme groups are a number of lines of different intensities which are not suppressed by self-induction, but which are greatly reduced in intensity.

The first spectrum of krypton (that of the simple uncondensed discharge), as given by Baly, terminates at λ 3500. This contains many lines which are not found in the second spectrum (that of the condensed discharge). The writer has found that in the region of the former, as given by Baly, the spectrum of the simple discharge includes apparently all the lines which survive with self-induction, as well as other lines which presumably belong to the first spectrum only. In the region beyond λ 3500 the spectrum of the simple discharge and that of the discharge with self-induction appear to be identical. The lines which survive with self-induction thus appear

in the spectra of all forms of discharge, and appear to be of a persistent type. It seems likely that if any numerical relationships exist, such as constant differences of wave-number, they might best be looked for among these persistent lines. A number of such approximately constant differences were actually found, and are given in Table V. The wave-lengths given in the first column do

TABLE IV

λ	C.	λ	C.	λ	C.
2483.71	9	2696.71	6	3191.33	8
2486.40	8	2697.41	6	3220.76	6
2487.75	6	2772.73	6	3224.99	6
2494.10	9	2817.00	8	3240.55	10
2497.81	10	2839.92	6	*3245.82	10
2511.83	6	2844.59	7	*3264.94	8
2513.03	6	2851.29	7	3268.61	7
2515.50	7	2870.73	8	3271.77	6
2519.38	6	2892.30	7	3286.01	7
2525.07	7	2893.81	6	3311.59	7
2528.51	6	2900.19	6	*3325.84	8
2538.43	7	2909.30	6	3330.88	8
2554.35	6	2952.69	8	3342.59	8
2555.23	6	2968.44	6	3352.07	10
2563.32	8	2992.36	9	3260.22	6
2565.72	6	3022.43	8	3375.09	6
2604.59	7	3024.57	9	3389.06	6
2630.76	7	3047.07	8	3396.72	6
*2639.86	10	*3056.86	9	*3439.60	9
2643.18	6	3063.26	8	3474.79	7
2648.26	6	3097.27	10	*3497.29	6
2670.78	7	3112.36	8	*3507.58	10
2680.44	7	3120.73	6	3514.68	6
2681.29	8	3124.52	7	*3549.57	8
2683.66	6	3141.48	10	3564.38	8
2690.35	7	3171.06	6	3641.48	6
2695.81	6	3189.23	8	3690.80	8

not belong to lines actually observed, but are strong lines in Baly's table. No attempt was made to ascertain the effect of self-induction upon them, on account of difficulties of identification in this region; but the constant differences between them and groups of persistent lines actually observed indicate probable relationship. The wave-numbers of these lines are given in the second column, and are followed by successive differences between the wave-numbers of lines belonging to each group. The actual discrepancies exceed the limits of accuracy of measurements, and this gives ground

for some hesitation; but the constant recurrence of differences so near to 95, 130, 165, and 722 seems significant when it is considered that account is taken of nearly all the strong persistent lines, and not arbitrarily selected groups. There are also a number of repetitions of one or another of these differences, or two of them, between isolated pairs or triplets.

TABLE V

λ	n	Δ_1	Δ_2	Δ_3	Δ_4
4059.02..........	2463.66	95.26	129.68	164.89	722.25
3947.76..........	2533.08	95.68	131.33	164.51	721.98
3920.29..........	2550.80	95.94	131.00	....	885.83 (164.50+721.33)
3906.37..........	2586.65	....	130.36	164.69	722.30
3850.23..........	2597.24	94.41	129.46	164.32	721.84
3836.64..........	2606.45	....	130.19	164.72	722.93
3817.23..........	2619.70	....	129.35	164.51	721.31
3783.28..........	2643.21	....	128.39	165.32	721.85
3754.35..........	2663.58	96.01	129.38	....	886.09 (164.50+721.59)

It is possible that further search in the region of greater wavelengths might reveal other lines having similar relationships; and they might also be found among the lines suppressed by self-induction. But the labor involved in such a search seems to be hardly justified in our present ignorance regarding the significance of such relationships.

PHYSICAL LABORATORY
UNIVERSITY OF CALIFORNIA
October 10, 1915

THE PRINCIPAL SERIES OF SODIUM

By R. W. WOOD and R. FORTRAT

In a paper published by one of us[1] in this *Journal*, it has been shown that the principal series of sodium is much more complete in the absorption than in the emission spectrum. But seven members of the series had been observed up to the time of the publication of this paper. The absorption method raised the number to forty-eight, a greater number than that found even in the case of chromospheric and stellar hydrogen. In this paper the opinion was expressed that higher resolving powers would bring more lines into view, since the head of the band, though clearly indicated on the photographic plate, was not resolvable into lines.

In the present paper we shall give the results of a repetition of this work made in the laboratory of Professor P. Weiss, at Zurich, with the largest and most powerful quartz spectrograph in the world, which he most kindly placed at our disposal. This instrument is designed for double transmission, through a train of six large quartz 60° prisms and one 30° prism backed by mercury (the equivalent of 13 prisms). In the vicinity of the head of the sodium series ($\lambda = 2414$) it is capable of resolving lines separated by only 0.03 A and has a dispersion such that one angstrom is represented by 3.5 mm on the plate. This enormous dispersion and separating power, while advantageous for the observation of lines, is very unfavorable for the study of variations of intensity along the continuous spectrum, and it is perhaps on this account that we have failed to detect the sudden decrease in the transmitting power of the vapor after the head of the band has been passed, which was recorded in the earlier paper.

The results which have been accomplished in the repetition of the work may be briefly summarized as follows: (*a*) The number of members of the series has been raised from forty-eight to fifty-eight, ten new lines at the head coming into view, and we are of the

[1] R. W. Wood, *Astrophysical Journal*, **29**, 97, 1909.

opinion that this marks the limit and that no further modification of the experiment or improvement of apparatus will enable any notable advance to be made. Possibly the use of a very long tube (say 15 or 20 m) would enable us to record a few more lines. (*b*) Previous investigations have shown as *doublets* only the first three members of the series, namely, the D lines, and the ultra-violet lines at $\lambda\lambda$ 3302 and 2853. We have distinctly resolved four more, so that we now have the wave-lengths of the components of the first seven doublets of the series. (*c*) The wave-lengths of all the lines of the series have been determined to the third place of the decimals with reference to the new secondary standards of Fabry and Buisson.

APPARATUS AND METHODS

The metallic sodium was vaporized in a tube of steel 2.8 m in length and 4 cm in diameter, heated electrically by a spiral of nickel wire insulated with asbestos and wound directly on the metal tube. The tube thus wound was packed in a thick layer of infusorial earth contained in a long wooden box; this prevented loss of heat, and enabled us to keep the tube at a red heat with an expenditure of current of about 1.3 kw. The ends of the tube, which protruded from the box, were closed with thin quartz lenses attached with sealing-wax, and kept cool by means of coils of lead tubing (soldered around the ends of the steel tube) through which a current of water circulated. Previous to heating, the tube was exhausted by a Gaede pump, and the pressure kept at a few millimeters throughout the experiment, by the occasional operation of the pump, which was necessary on account of the evolution of hydrogen by the sodium. As a source of light we required something which would give us a continuous spectrum of great intensity in the ultra-violet. The spark under water was first tried, but we finally adopted a quartz mercury arc of special construction which was placed at our disposal by Professor A. Cotton of Paris. In this lamp the arc passes through a heavy tube of quartz 3 cm in length and 4 mm in diameter. The lamp is constructed in such a way that the light from the narrow tube (end-on) passes out through a window of optically worked, fused quartz, and it operates at

a full red heat.[1] The spectrum is quite different from that given out by the usual type of quartz mercury arc, for the lines are enormously broadened and only slightly brighter than the continuous background.

This lamp was mounted end-on at the principal focus of the first quartz lens (for the wave-length in question) and the parallel beam, after traversing the tube, was focused on the slit of the spectrograph. The temperature of the tube and the resulting vapor-density of the sodium were regulated according to the portion of the series under investigation. The D lines probably appear when the temperature is as low as 100° C. (the senior writer has seen them with an echelon with a glass bulb of vapor only 8 cm in diameter at a temperature of 120°), while to get the last members near the head of the bands a density corresponding to a red heat is necessary. One sacrifices the sharpness of the lines somewhat by this augmentation of density, and it is possible that slightly better results could be obtained with a longer tube (say 20 m) at a lower temperature. With a little practice one soon learns how to control the temperature for any portion of the series by observing the width of the D lines, and the condition of the channeled spectra in the red and green, in the transmitted light, by means of a small direct-vision spectroscope.

RESULTS

To secure the complete spectrum, it was of course necessary to expose a large number of plates, on account of the enormous dispersion of the spectrograph. In each case a comparison spectrum of iron was impressed on the plate in coincidence with the absorption spectrum. The values of λ for the D lines are those obtained by the recent measurements by means of interference fringes (*étalons secondaires* of Buisson and Fabry). For the lines at λ 3302 we have taken the values given by Kayser and Runge, correcting them by comparing their values for the iron lines with the new standards of Buisson and Fabry. For the value of the lines at λ 2853 we have taken that of Fortrat for the emission lines,

[1] The lamp, "type Pistolet," was constructed by the Westinghouse Cooper-Hewitt Company, Paris Branch.

determined with reference to the same standards. The other lines were determined from the photographs of the absorption spectrum. The wave-lengths, correct to a few thousandths of an angstrom unit, are given in Table I.

REGULARITY OF THE SERIES

It is well known that Ritz has given a formula for the principal and subordinate series which is closely related to the formula of Balmer, which expresses the frequency ν as follows:

$$\nu=N\left(\frac{1}{4}-\frac{1}{m^2}\right).$$

Though this formula represents the series of hydrogen very exactly ($N=109{,}678.4$ and $m=3, 4, 5, \ldots\ldots, 32$), no other series has been found which is well represented by so simple an expression.

The formula which Ritz has proposed for the principal series[1] is the following:

$$\nu=N\left[\frac{1}{A^2}-\frac{1}{\left(m+a-\frac{b}{m^2}\right)^2}\right],$$

in which N is the universal constant 109,674.8, A, a, and b are constants depending on the element which emits the series, and m takes the successive values 2, 3, 4, ... , etc.

The form of A is an essential point in the theory.

$$\frac{1}{A^2}=\frac{1}{\left(1.5+a'+\frac{b'}{(1.5)^2}\right)^2}.$$

The principal series of sodium is a series of doublets, the two components of the successive members approaching as the wave-length decreases. In the Ritz formula there are two values for each of the constants a and b, but single values only for the constants a' and b'. For the calculation of the constants we have employed the three first doublets of the series, i.e., the D lines and the ultra-violet lines at wave-lengths 3302 and 2853, reducing them to the new standards as previously explained.

[1] *Collected Works*, pp. 52 and 100; *Annalen der Physik*, 25, 662, 1908.

TABLE I

m	Observed Wave-Length	Observed Frequency	Difference between Calculated Frequency and Observed Frequency
2 D_1	5895.930	16,955.88	0.00
2 D_2	5889.963	16,973.02	−0.04
3 D_1	3302.94	30,266.66	−2.80
3 D_2	3302.34	30,271.10	−1.29
4 D_1	2853.031	35,039.67	0.00
4 D_2	2852.828	35,043.10	+0.94
5 D_1	2680.443	37,295.71	+0.38
5 D_2	2680.335	37,297.67	−0.45
6 D_1	2593.927	38,539.68	+0.27
6 D_2	2593.828	38,541.07	+0.07
7 D_1	2543.875	39,297.86	+0.21
7 D_2	2543.817	39,298.76	+0.03
8 D_1	2512.210	39,793.16	+0.97
8 D_2	2512.128	39,794.46	+0.15
9 D_2	2490.733	40,136.27	+0.36
10 D_2	2475.533	40,382.73	−0.04
11	2464.397	40,565.19	+0.25
12	2455.915	40,705.28	−0.36
13	2449.393	40,813.65	+0.13
14	2444.195	40,900.45	−0.07
15	2440.046	40,970.02	+0.37
16	2436.627	41,027.48	+0.43
17	2433.824	41,074.69	+0.77
18	2431.433	41,115.11	+0.35
19	2429.428	41,149.05	+0.30
20	2427.705	41,178.24	+0.09
21	2426.217	41,203.51	−0.23
22	2424.937	41,225.25	−0.31
23	2423.838	41,243.90	−0.24
24	2422.856	41,260.61	−0.16
25	2421.997	41,275.28	−0.16
26	2421.233	41,288.29	−0.15
27	2420.520	41,300.45	−0.71
28	2419.922	41,310.67	−0.54
29	2419.380	41,319.92	−0.45
30	2418.893	41,328.25	−0.35
31	2418.454	41,335.72	−0.20
32	2418.062	41,342.43	+0.02
33	2417.695	41,348.70	+0.06
34	2417.362	41,354.41	+0.11
35	2417.058	41,359.60	+0.20
36	2416.779	41,364.39	+0.25
37	2416.518	41,368.87	+0.23
38	2416.271	41,373.08	+0.16
39	2416.046	41,376.94	+0.08
40	2415.838	41,380.51	+0.03

TABLE I—*Continued*

m	Observed Wave-Length	Observed Frequency	Difference between Calculated Frequency and Observed Frequency
41........	2415.651	41,383.70	+0.10
42........	2415.474	41,386.74	+0.10
43........	2415.305	41,389.64	+0.03
44........	2415.147	41,392.34	−0.03
45........	2415.006	41,394.77	+0.01
46........	2414.872	41,397.07	+0.01
47........	2414.746	41,399.22	+0.03
48........	2414.627	41,401.27	0.00
49........	2414.518	41,403.13	+0.05
50........	2414.411	41,404.97	0.00
51........	2414.313	41,406.65	+0.01
52........	2414.218	41,408.28	−0.03
53........	2414.131	41,409.77	−0.05
54........	2414.050	41,411.16	+0.02
55........	2413.971	41,412.52	+0.01
56........	2413.910	41,413.57	+0.23
57........	2413.873	41,414.20	+0.81
58........	2413.837	41,414.81	+1.34

For the lines below the last of the resolvable doublets ($\lambda\lambda$ 2512.210 and 2512.128) we have compared the measured values with the values calculated from the constants of the D_2 series, since the line D_2 is more intense than D_1. For D_1 we found:

$$\nu=41,448.59-\frac{109,678.4}{\left(m+0.14373-\frac{0.11043}{m^2}\right)^2}\,.$$

For D_2:

$$\nu=41,448.59-\frac{109,678.4}{\left(m+0.14497-\frac{0.11240}{m^2}\right)^2}\,.$$

In the last column of Table I will be found the differences between the calculated and observed frequencies. The calculated position of the head of the series is

$$\lambda=2412.627,$$

and we have therefore reached a point only 1.21 A from the theoretical end.

The last thirteen members of the series are plotted from the wave-length determinations in Fig. 1. The calculated head of the series lies to the left of the fifty-eighth line at a distance equal to the length of the portion of the series figured (i.e., the distance between the forty-sixth and fifty-eighth members). If we continue plotting the series on the same scale, the D lines will lie on our right at a distance of 348.3 m, or more than a thousand feet! The last thirty-two members, or more than half of the entire series, cover a spectral range not wider than the distance between the D lines.

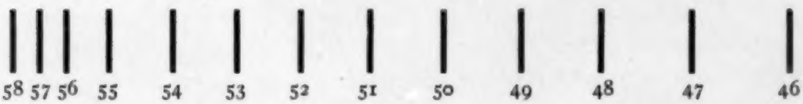


Fig. 1

If we examine the last column of the table, which gives the differences between the measured and calculated frequencies, we see that the formula of Ritz represents the series with a surprising degree of accuracy. The *differences* of frequency become much less when we convert them into differences of wave-length. For example, in the case of the last line measured ($m=58$), which could not be very accurately determined on account of its faintness, the difference amounts to only 0.08 of an angstrom. This is the largest difference except that for $m=3$, which is -2.8, and Ritz has already pointed out the disagreement which occurs at the *beginning* of the series, which seems to be a general phenomenon.

The disagreement which we have found at the *end* of the series may result from slight errors of measurement. Neglecting the first and last three members, the differences vary in a systematic manner and indicate the degree of imperfection of the formula. In the case of the largest, where the error of measurement probably adds itself to the error of formula, the difference is less than 0.05 A.

As is well known, only the first seven of these sixty-eight members have been found in the emission spectrum. The explanation of this circumstance seems to be not difficult. In the first place, Ritz has supposed that "turbulent motions," such as occur in the spark, are unfavorable to the appearance of the higher

members, i.e., more lines appear in the arc than in the spark. On this hypothesis we should expect the comparatively cool and quiescent vapor used in absorption experiments to be particularly favorable to the development of the groupings that give the higher members (Ritz's theory of magnetic atoms). Moreover, if we assume that the groupings which form the higher members occur only in small proportion, we can bring them out only by using an immense volume of vapor, which can be done to better advantage by the absorption method.

ZURICH
1914[1]

[1] Publication delayed on account of the war.

Minor Contributions and Notes

NOTES ON THE USE OF THE CONCAVE GRATING

THE EFFECT OF TEMPERATURE ON THE CONCAVE GRATING

All investigators agree that in order to get perfect definition with a concave grating, the temperature during the exposure must be constant, barometric pressure being assumed constant. However, there still appears to be some uncertainty as to the exact nature and extent of the effects of varying temperature.

The author,[1] after a rather careful investigation, convinced himself that the only important effect of changing temperature is the change in the width of the grating space. The dispersion is directly proportional to this width, and hence also the distance of all lines from the slit. It follows immediately from this that on any one plate the shift of all lines is approximately the same. It is to this point that Grebe,[2] in an article on this same subject, devotes most of his attention. He deals entirely with the effect of temperature on the accuracy of wave-length measurement, and concludes with the statement that, even for the most accurate work, variations of temperature of not more than 1° C. are non-injurious.

This statement is not at all true when the definition of the grating is under discussion, a change of 1° C. shifting λ 5000 the relatively large amount of 0.13 A. This is several times the resolving power of a large grating. For band spectra, with which the author is primarily concerned, the best possible definition is essential, for even the largest gratings, used under the best conditions, have far too small dispersion and resolving power to separate many lines. For such work it is necessary to keep the temperature, after suitable correction for barometric pressure, within a range of 0°1 C., if one is to secure the best results.

[1] *Astrophysical Journal*, **39**, 55, 1914.

[2] *Zeitschrift für wissenschaftliche Photographie*, **13**, 264, 1914.

The author wishes, in addition, to call particular attention to another effect of temperature, which up to the present appears to have been overlooked. This is the effect upon the definition of *temperature-gradients* across the face of the grating. The magnitude of such gradients depends, of course, primarily upon the location of the grating. The effects which the author has observed, using the large concave grating of the University of Wisconsin, may be quite lacking on other gratings more favorably situated. This grating, as has been mentioned before (*loc. cit.*), is mounted inside a double-walled room built entirely within an ordinary room. The situation of the outer room is such, however, that almost the entire flow of heat takes place naturally through the west side, and unfortunately the rail carrying the grating carriage is located close to this side. Even when no attempt is made to control temperature, the grating's temperature never changes at a rate of more than 0°1 per hour. The effect of such a change on short exposures is negligible. But the transfer of heat entirely through the west wall sets up temperature-gradients in the grating which have been found experimentally to produce a loss of definition equal to that caused by a direct change of temperature of fully 1° C. This effect is so serious that the author made a careful investigation to convince himself that it was really due to temperature-gradients. Even after the temperature becomes constant, it is necessary to wait for one or two hours, before these gradients are entirely "smoothed out."

In conclusion, then, I desire to repeat that in order to obtain the best definition with gratings subject to unequal heating from different sides, it is necessary, in the case of *short* as well as of long exposures, for the grating to have a constant temperature not only during the exposure but for an hour or two previous to that time.

THE EFFECT OF TEMPERATURE UPON THE COINCIDENCE OF ORDERS OF THE CONCAVE GRATING

Miss Howell[1] has shown, for the particular pair of lines λ 5640 and λ 2820, that there is a shift of 0.005 A in the coincidence of orders of a concave grating, when the temperature is varied from

[1] *Astrophysical Journal*, 39, 230, 1914.

0° to 10° C. She does not, however, explain why Kayser[1] found no such shift (15°–25°), nor does she show how the shift varies with wave-length. Instead, she advises reducing all wave-lengths to standard conditions (15° C. and 760 mm) when using the coincidence of orders. This, however, involves a considerable amount of quite unnecessary work.

Using the ordinary formula for change of index of refraction with temperature:

$$n_t - 1 = \frac{n_0 - 1}{1 + at}, \tag{1}$$

where

n_t = index of refraction at t° C.
n_0 = index of refraction at 0° C.
a = 0.00367 (air)
t = temperature Centigrade

it can easily be shown that if any two lines coincide at t_1, the amount of non-coincidence (i.e., shift) at any other temperature t_2 is

$$x = \frac{\lambda_1 a(t_2 - t_1)\Delta n}{1 + a(t_2 + t_1)}, \tag{2}$$

where

Δn = difference of the indices of refraction of the two lines at 0° C. (a *negative* quantity)
λ_1 = line of longer wave-length (i.e., smaller order)
x = shift from coincidence in terms of λ_1

In deriving this expression only terms of the first order were retained. It is, however, quite accurate for all practical conditions.

Formula (2) shows that there will always be a shift in coincidence, with changing temperature, if air has any dispersion (Δn) whatever. Kayser, computing the shift by the longer direct method, evidently failed to carry out a sufficient number of places in the calculations, and so got in some cases a negative result.

Δn depends upon the curve of dispersion of air. The latest work on this is by Miss Howell.[2] She also gives the results of all previous investigators (*op. cit.*, p. 87). These different results give

[1] *Handbuch der Spectroscopie*, **1**, 720.

[2] *Physical Review* (2), **6**, 81, 1915.

different values of Δn, but the difference is not great enough to affect the work appreciably.

As an example of the use of formula (2) I take Miss Howell's figures (1):

λ 5640 and λ 2820 coincide at 0° C. What is the shift at 10°? Here $\lambda_1=5640$, $a=0.00367$, $t_2-t_1=10°$, $\Delta n=1.0002924-1.0003091=-0.0000167$, $t_2+t_1=10°$.

$\therefore x=-0.00334$ A. That is, λ 5640 will fall 0.003 A to the *violet* of λ 2820. Miss Howell gives 0.005 A, showing again how easily large errors can creep into a result in a calculation involving small differences of large quantities, such as she uses.

It may be said in general that, for second- and third-order coincidence, the shift is negligible above $\lambda_1=4500$ and $t_2-t_1<5°$. For the second- and first-order coincidence the shift is always more than 0.001 A for $t_2-t_1>5°$. This shows that, for summer temperatures of 10° or more above the standard 15° C., it is necessary to make a correction for the effect of temperature upon the coincidence of orders.

ABSORBING SOLUTIONS FOR THE CONCAVE GRATING

In using an absorbing solution to cut out certain orders of a concave grating, the spectroscopist desires a solution as transparent as possible in one definite region, and as opaque as possible in another. The photographs and accompanying description in atlases of absorption spectra are not usually explicit enough in this regard. The author in some cases has found 10 per cent transmission at a wave-length apparently within the region of total absorption, while 1 per cent to 2 per cent transmission is common. The following figures may therefore be of use to other spectroscopists. It is hoped that they are accurate enough for all practical purposes. The solutions given, with the addition of glass, are sufficient for most ordinary work on the grating.

1. $CuCl_2$, aqueous solution, $2\times$normal:
   No absorption λ 4380–6000
   Increasing absorption to red
   Transmission 40 per cent at λ 4110

Transmission 6 per cent at λ 3950
Transmission 2 per cent at λ 3800
Transmission 0.2 per cent at λ 3590
Absorption increasing rapidly at λ 3590

2. K_2CrO_4, aqueous solution, $\frac{1}{2}\times$normal:
No absorption above λ 5100
Transmission 10 per cent at λ 4880
Transmission 1 per cent at λ 4750
Transmission less than 0.1 per cent at λ 4500
Transmission less than 0.02 per cent at λ 3500–4000
Transmission less than 1 per cent at λ 3000–3400

(NOTE.—All the solutions were contained in the ordinary glass U-shaped absorption cell, with parallel sides. Soret[1] gives for K_2CrO_4 a region of comparative transparency at λ 3000–3300. In this region the iron lines are relatively faint and 1 per cent is the best ratio that I have.)

3. $K_4Cr_2O_7$, aqueous solution, $\frac{1}{16}\times$normal:
No absorption above λ 5450
Transmission 10 per cent at λ 5120
Transmission less than 0.5 per cent at λ 4900
Transmission less than 0.05 per cent at λ 4400
Transmission less than 0.1 per cent at λ 3400–3900
Transmission less than 1 per cent at λ 3000–3400

The foregoing results were obtained with exposures of different lengths of the iron arc. I used the Pfund type, with one electrode carbon, an arrangement giving an arc of extremely constant intensity, with respect both to time and to the particular area used.

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[1] *Arch. sc. phys. et nat.* (2), **61**, 322, 1878.

Reviews

X-Rays. An Introduction to the Study of Röntgen Rays. By C. W. C. Kaye. London and New York: Longmans, Green, & Co., 1914. Pp. xix+251. $1.25.

This is a most delightful and unpretentious little book which contains much more sound learning than the modesty of the author permits him to claim for it. Indeed I know of no other book which can take its place either for the student of pure physics or for the practical X-ray worker. There are other books on X-rays, but no others which condense into so small a compass so comprehensive and intelligible a summary of what is now known of the subject. Approximately the first half of the book is concerned with the simple statement of the most important of the facts having to do with the technique of X-ray production. Then come the chapters which are all-important for the theorist. The most notable of these are chap. ix, on "Secondary Rays," pp. 108–144, and chap. xii, pp. 168–205, on "Interference and Reflection of X-Rays." These chapters contain an illuminating and able account of the three great discoveries of the past decade: (1) the discovery by Barkla of characteristic X-rays, (2) the discovery by Laue of the crystal grating, and (3) the discovery by Moseley of the relations between the wave-lengths of the characteristic X-rays of the different elements. Although the subject treated is growing rapidly, Kaye's little book will not quickly be outgrown.

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An Introduction to the Study of Variable Stars. By Caroline E. Furness. Boston: Houghton Mifflin Co., 1915. 12mo, pp. xv+327, with 14 plates and 36 figures in the text. $1.75 net (weight 30 oz.).

The phenomena of stellar variation are taking on new astrophysical meanings from recent investigations and discussions, and offer unexcelled opportunities for exploring cosmic mysteries. The data for these discussions have been furnished by amateur observers in greater proportion than in any other important branch of astronomy, yet there has been no

comprehensive book to instruct the observer and warn him against pitfalls. The present work, which is one of the volumes issued in commemoration of the semi-centennial of Vassar College, meets this need very completely. The author's experience as a teacher and as director of the Vassar College Observatory is shown in the quality of the book. In addition to the historical and practical treatment, descriptions of underlying principles and also of the latest methods and instruments which have been used in astrophysical investigations of the phenomena of stellar variation are given in considerable detail.

The first chapter is introductory, and gives a bird's-eye view of the whole subject. The following fourteen chapters go into details and fall naturally into four divisions: (1) the observer's equipment (chaps. ii–v), maps, charts, catalogues, meaning of the term magnitude (this corresponds to Part I in Hagen's work); (2) photometry of variables (chaps. vi–viii), visual, photographic, photo-electric; (3) use of the observations (chaps. ix–xi), light-scale, light-curves, elements of variation, and predictions from elements; (4) deductions from the data in the preceding chapters (chaps. xii–xv), eclipsing binary variables, long-period variables, statistics of stellar variation, hints to observers, tables.

The professional as well as the amateur observer will read with profit the practical hints given in chap. xv on the use of the telescope, time, identification of variables, methods of recording, and precautions. The author shows herself to be possessed of the spirit of both the teacher and investigator, in thus combining the description of underlying principles with that of the latest extensions of work on variable stars.

From the standpoint of the astrophysicist a few points would seem open to modification; for example, the failure in chap. i to give credit to Mrs. Fleming for the Harvard classification of spectra. The Zöllner photometer is described on p. 118 as provided with the historical petroleum lamp instead of the incandescent electric lamp which is now used. On p. 141 halation circles on the photographs are spoken of as unavoidable. On p. 142, in the description of Fig. 21, "Color-Curve of the 40-Inch Objective," the larger ordinates are made to lie in the wrong direction. On p. 221, in the description of the correcting lens, the statement is made that "such a lens is always necessary when the telescope which is intended for visual work is used for photographic purposes," while in fact such a correcting lens is useful only with the spectroscope. No mention is made of the important fact that most classes of variables show a change in color along with the change in light, in the direction of increasing color-intensity with decreasing light. This is

perhaps the only important factor in the modern theory of stellar variation that has been omitted by the author.

The press work, binding, and illustrations are very good; but the plates of stellar spectra, Nos. II, X, XI, XIII, and XIV, show no wavelengths or anything to indicate the range included; also Plate XI is printed with the violet end to the right instead of in the usual direction. Plate II, typical spectra, is printed as a negative, while the rest show as positives. It would seem better that all should be positives.

J. A. P.

Collected List of Lunar Formations. Compiled by MARY A. BLAGG under the direction of the late S. A. SAUNDER. Published by the Lunar Nomenclature Committee of the International Association of Academies. Edinburgh, 1913. 8vo. Pp. 182.

A recent example of successful international co-operation in scientific work is furnished by the Lunar Nomenclature Committee of the International Association of Academies. The Committee, which originally consisted of Loewy, Franz, Newcomb, Saunder, Weiss, and Turner, suffered greatly by the death of members especially interested in the moon, but the plans laid down are largely completed.

Measures for an accurate map of the moon were made by Franz at Breslau and Saunder in England. The drawing of the central portions has been nearly completed in England by W. H. Wesley, and it is expected that the outer portions will yet be drawn in Breslau.

A collection of the names of lunar formations was planned by Saunder and has been carefully completed by Miss Blagg. The funds for printing the list were provided by the Paris Academy of Sciences. The work is a long step in the problem of lunar nomenclature and will be the basis from which a uniform system may be devised.

The list gives in parallel columns the names given by Neison, Schmidt, and Beer and Mädler, with references, together with the descriptive position and nature of the object. At the end of the list are many pages of useful notes concerning the individual formations. There are 4,789 numbers in the list, which includes practically every lunar feature. The list will prove most useful to observers of the moon, for it allows the different authorities to be compared with certainty and without delay.

A. H. JOY